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9.1 Introduction

A rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called *shank* or *body* and lower portion of shank is known as *tail*, as shown in Fig. 9.1. The rivets are used to make permanent fastening between the plates such as in structural work, ship building, bridges, tanks and boiler shells. The riveted joints are widely used for joining light metals.

The fastenings (*i.e.* joints) may be classified into the following two groups:

1. Permanent fastenings, and
2. Temporary or detachable fastenings.
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The permanent fastenings are those fastenings which can not be disassembled without destroying the connecting components. The examples of permanent fastenings in order of strength are soldered, brazed, welded and riveted joints.

The temporary or detachable fastenings are those fastenings which can be disassembled without destroying the connecting components. The examples of temporary fastenings are screwed, keys, cotters, pins and splined joints.

9.2 Methods of Riveting

The function of rivets in a joint is to make a connection that has strength and tightness. The strength is necessary to prevent failure of the joint. The tightness is necessary in order to contribute to strength and to prevent leakage as in a boiler or in a ship hull.

When two plates are to be fastened together by a rivet as shown in Fig. 9.2 (a), the holes in the plates are punched and reamed or drilled. Punching is the cheapest method and is used for relatively thin plates and in structural work. Since punching injures the material around the hole, therefore drilling is used in most pressure-vessel work. In structural and pressure vessel riveting, the diameter of the rivet hole is usually 1.5 mm larger than the nominal diameter of the rivet.

The plates are drilled together and then separated to remove any burrs or chips so as to have a tight flush joint between the plates. A cold rivet or a red hot rivet is introduced into the plates and the point (i.e. second head) is then formed. When a cold rivet is used, the process is known as cold riveting and when a hot rivet is used, the process is known as hot riveting. The cold riveting process is used for structural joints while hot riveting is used to make leak proof joints.

A ship’s body is a combination of riveted, screwed and welded joints.
The riveting may be done by hand or by a riveting machine. In hand riveting, the original rivet head is backed up by a hammer or heavy bar and then the die or set, as shown in Fig. 9.2 (a), is placed against the end to be headed and the blows are applied by a hammer. This causes the shank to expand thus filling the hole and the tail is converted into a point as shown in Fig. 9.2 (b). As the rivet cools, it tends to contract. The lateral contraction will be slight, but there will be a longitudinal tension introduced in the rivet which holds the plates firmly together.

In machine riveting, the die is a part of the hammer which is operated by air, hydraulic or steam pressure.

Notes: 1. For steel rivets up to 12 mm diameter, the cold riveting process may be used while for larger diameter rivets, hot riveting process is used.
2. In case of long rivets, only the tail is heated and not the whole shank.

9.3 Material of Rivets

The material of the rivets must be tough and ductile. They are usually made of steel (low carbon steel or nickel steel), brass, aluminium or copper, but when strength and a fluid tight joint is the main consideration, then the steel rivets are used.

The rivets for general purposes shall be manufactured from steel conforming to the following Indian Standards:

(a) IS : 1148–1982 (Reaffirmed 1992) – Specification for hot rolled rivet bars (up to 40 mm diameter) for structural purposes; or


9.4 Essential Qualities of a Rivet

According to Indian standard, IS : 2998 – 1982 (Reaffirmed 1992), the material of a rivet must have a tensile strength not less than 40 N/mm² and elongation not less than 26 percent. The material must be of such quality that when in cold condition, the shank shall be bent on itself through 180° without cracking and after being heated to 650°C and quenched, it must pass the same test. The rivet when hot must flatten without cracking to a diameter 2.5 times the diameter of shank.

9.5 Manufacture of Rivets

According to Indian standard specifications, the rivets may be made either by cold heading or by hot forging. If rivets are made by the cold heading process, they shall subsequently be adequately heat treated so that the stresses set up in the cold heading process are eliminated. If they are made by hot forging process, care shall be taken to see that the finished rivets cool gradually.

9.6 Types of Rivet Heads

According to Indian standard specifications, the rivet heads are classified into the following three types:

1. Rivet heads for general purposes (below 12 mm diameter) as shown in Fig. 9.3, according to IS : 2155 – 1982 (Reaffirmed 1996).
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Fig. 9.3. Rivet heads for general purposes (below 12 mm diameter).

2. Rivet heads for general purposes (From 12 mm to 48 mm diameter) as shown in Fig. 9.4, according to IS : 1929 – 1982 (Reaffirmed 1996).

Fig. 9.4. Rivet heads for general purposes (from 12 mm to 48 mm diameter)
3. Rivet heads for boiler work (from 12 mm to 48 mm diameter, as shown in Fig. 9.5, according to IS: 1928 – 1961 (Reaffirmed 1996).

![Fig. 9.5. Rivet heads for boiler work.](image)

The **snap heads** are usually employed for structural work and machine riveting. The **counter sunk heads** are mainly used for ship building where flush surfaces are necessary. The **conical heads** (also known as **conoidal heads**) are mainly used in case of hand hammering. The **pan heads** have maximum strength, but these are difficult to shape.

### 9.7 Types of Riveted Joints

Following are the two types of riveted joints, depending upon the way in which the plates are connected.

1. Lap joint, and 2. Butt joint.
A lap joint is that in which one plate overlaps the other and the two plates are then riveted together.

A butt joint is that in which the main plates are kept in alignment butting (i.e. touching) each other and a cover plate (i.e. strap) is placed either on one side or on both sides of the main plates. The cover plate is then riveted together with the main plates. Butt joints are of the following two types:


In a single strap butt joint, the edges of the main plates butt against each other and only one cover plate is placed on one side of the main plates and then riveted together.

In a double strap butt joint, the edges of the main plates butt against each other and two cover plates are placed on both sides of the main plates and then riveted together.

In addition to the above, following are the types of riveted joints depending upon the number of rows of the rivets.


A single riveted joint is that in which there is a single row of rivets in a lap joint as shown in Fig. 9.6 (a) and there is a single row of rivets on each side in a butt joint as shown in Fig. 9.8.

A double riveted joint is that in which there are two rows of rivets in a lap joint as shown in Fig. 9.6 (b) and (c) and there are two rows of rivets on each side in a butt joint as shown in Fig. 9.9.

Similarly the joints may be triple riveted or quadruple riveted.

Notes: 1. When the rivets in the various rows are opposite to each other, as shown in Fig. 9.6 (b), then the joint is said to be chain riveted. On the other hand, if the rivets in the adjacent rows are staggered in such a way that
every rivet is in the middle of the two rivets of the opposite row as shown in Fig. 9.6 (c), then the joint is said to be zig-zag riveted.

2. Since the plates overlap in lap joints, therefore the force $P, P$ acting on the plates [See Fig. 9.15 (a)] are not in the same straight line but they are at a distance equal to the thickness of the plate. These forces will form a couple which may bend the joint. Hence the lap joints may be used only where small loads are to be transmitted. On the other hand, the forces $P, P$ in a butt joint [See Fig. 9.15 (b)] act in the same straight line, therefore there will be no couple. Hence the butt joints are used where heavy loads are to be transmitted.

![Fig. 9.7. Triple riveted lap joint.](image)

![Fig. 9.8. Single riveted double strap butt joint.](image)
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Fig. 9.9. Double riveted double strap (equal) butt joints.

(a) Chain riveting.  (b) Zig-zag riveting.

Fig. 9.10. Double riveted double strap (unequal) butt joint with zig-zag riveting.

9.10 Important Terms Used in Riveted Joints

The following terms in connection with the riveted joints are important from the subject point of view:

1. **Pitch.** It is the distance from the centre of one rivet to the centre of the next rivet measured parallel to the seam as shown in Fig. 9.6. It is usually denoted by $p$.

2. **Back pitch.** It is the perpendicular distance between the centre lines of the successive rows as shown in Fig. 9.6. It is usually denoted by $p_b$.

3. **Diagonal pitch.** It is the distance between the centres of the rivets in adjacent rows of zig-zag riveted joint as shown in Fig. 9.6. It is usually denoted by $p_d$.

4. **Margin or marginal pitch.** It is the distance between the centre of rivet hole to the nearest edge of the plate as shown in Fig. 9.6. It is usually denoted by $m$. 
In order to make the joints leak proof or fluid tight in pressure vessels like steam boilers, air receivers and tanks etc. a process known as caulking is employed. In this process, a narrow blunt tool called caulking tool, about 5 mm thick and 38 mm in breadth, is used. The edge of the tool is ground to an angle of 80°. The tool is moved after each blow along the edge of the plate, which is planed to a bevel of 75° to 80° to facilitate the forcing down of edge. It is seen that the tool burrs down the plate at point A in Fig. 9.12 (a) forming a metal to metal joint. In actual practice, both the edges at A and C are caulked.

9.11 Caulking and Fullering

Caulking and fullering are processes used to make riveted joints leak proof or fluid tight in steam boiler.
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*B* are caulked. The head of the rivets as shown at *C* are also turned down with a caulkling tool to make a joint steam tight. A great care is taken to prevent injury to the plate below the tool.

A more satisfactory way of making the joints staunch is known as *fullering* which has largely superseded caulking. In this case, a fullering tool with a thickness at the end equal to that of the plate is used in such a way that the greatest pressure due to the blows occur near the joint, giving a clean finish, with less risk of damaging the plate. A fullering process is shown in Fig. 9.12 (b).

**9.12 Failures of a Riveted Joint**

A riveted joint may fail in the following ways:

1. **Tearing of the plate at an edge.** A joint may fail due to tearing of the plate at an edge as shown in Fig. 9.13. This can be avoided by keeping the margin, *m* = 1.5d, where *d* is the diameter of the rivet hole.

2. **Tearing of the plate across a row of rivets.** Due to the tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets as shown in Fig. 9.14. In such cases, we consider only one pitch length of the plate, since every rivet is responsible for that much length of the plate only.

   The resistance offered by the plate against tearing is known as tearing resistance or tearing strength or tearing value of the plate.

   Let

   \[
   p = \text{Pitch of the rivets},
   \]

   \[
   d = \text{Diameter of the rivet hole},
   \]

   \[
   t = \text{Thickness of the plate, and}
   \]

   \[
   \sigma_t = \text{Permissible tensile stress for the plate material.}
   \]

   We know that tearing area per pitch length,

   \[
   A_t = (p - d) t
   \]

   ∴ Tearing resistance or pull required to tear off the plate per pitch length,

   \[
   P_t = A_t \sigma_t = (p - d) t \sigma_t
   \]

   When the tearing resistance (\(P_t\)) is greater than the applied load (\(P\)) per pitch length, then this type of failure will not occur.

3. **Shearing of the rivets.** The plates which are connected by the rivets exert tensile stress on the rivets, and if the rivets are unable to resist the stress, they are sheared off as shown in Fig. 9.15.
It may be noted that the rivets are in *single shear in a lap joint and in a single cover butt joint, as shown in Fig. 9.15. But the rivets are in double shear in a double cover butt joint as shown in Fig. 9.16. The resistance offered by a rivet to be sheared off is known as shearing resistance or shearing strength or shearing value of the rivet.

Let
\[ d = \text{Diameter of the rivet hole}, \]
\[ \tau = \text{Safe permissible shear stress for the rivet material}, \]
\[ n = \text{Number of rivets per pitch length}. \]

We know that shearing area,
\[ A_s = \frac{\pi}{4} \times d^2 \quad \text{...(In single shear)} \]
\[ = 2 \times \frac{\pi}{4} \times d^2 \quad \text{...(Theoretically, in double shear)} \]
\[ = 1.875 \times \frac{\pi}{4} \times d^2 \quad \text{...(In double shear, according to Indian Boiler Regulations)} \]

\[ \therefore \text{Shearing resistance or pull required to shear off the rivet per pitch length,} \]
\[ P_s = n \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{...(In single shear)} \]
\[ = n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{...(Theoretically, in double shear)} \]

* We have already discussed in Chapter 4 (Art. 4.8) that when the shearing takes place at one cross-section of the rivet, then the rivets are said to be in single shear. Similarly, when the shearing takes place at two cross-sections of the rivet, then the rivets are said to be in double shear.
When the shearing resistance \((P_s)\) is greater than the applied load \((P)\) per pitch length, then this type of failure will occur.

4. **Crushing of the plate or rivets.** Sometimes, the rivets do not actually shear off under the tensile stress, but are crushed as shown in Fig. 9.17. Due to this, the rivet hole becomes of an oval shape and hence the joint becomes loose. The failure of rivets in such a manner is also known as **bearing failure.** The area which resists this action is the projected area of the hole or rivet on diametral plane.

The resistance offered by a rivet to be crushed is known as **crushing resistance** or **crushing strength** or **bearing value** of the rivet.

Let 
\[ d = \text{Diameter of the rivet hole}, \]
\[ t = \text{Thickness of the plate}, \]
\[ \sigma_c = \text{Safe permissible crushing stress for the rivet or plate material, and} \]
\[ n = \text{Number of rivets per pitch length under crushing}. \]

We know that crushing area per rivet \((i.e. \text{projected area per rivet}), \)
\[ A_c = d.t \]
\[ \therefore \text{Total crushing area} = n.d.t \]
and crushing resistance or pull required to crush the rivet per pitch length,
\[ P_c = n.d.t.\sigma_c \]

When the crushing resistance \((P_c)\) is greater than the applied load \((P)\) per pitch length, then this type of failure will occur.

**Note:** The number of rivets under shear shall be equal to the number of rivets under crushing.

9.13 **Strength of a Riveted Joints**

The strength of a joint may be defined as the maximum force, which it can transmit, without causing it to fail. We have seen in Art. 9.12 that \(P_r, P_s\) and \(P_c\) are the pulls required to tear off the plate, shearing off the rivet and crushing off the rivet. A little consideration will show that if we go on increasing the pull on a riveted joint, it will fail when the least of these three pulls is reached, because a higher value of the other pulls will never reach since the joint has failed, either by tearing off the plate, shearing off the rivet or crushing off the rivet.

If the joint is **continuous** as in case of boilers, the strength is calculated **per pitch length.** But if the joint is **small,** the strength is calculated for the **whole length** of the plate.

9.14 **Efficiency of a Riveted Joint**

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to the strength of the un-riveted or solid plate.

We have already discussed that strength of the riveted joint
\[ = \text{Least of } P_r, P_s\text{ and } P_c \]
Strength of the un-riveted or solid plate per pitch length,
\[ P = p \times t \times \sigma_t \]
\[ \eta = \frac{\text{Least of } P, P_s, P_c}{p \times t \times \sigma_t} \]

where
\[ p = \text{Pitch of the rivets}, \]
\[ t = \text{Thickness of the plate}, \]
\[ \sigma_t = \text{Permissible tensile stress of the plate material}. \]

**Example 9.1.** A double riveted lap joint is made between 15 mm thick plates. The rivet diameter and pitch are 25 mm and 75 mm respectively. If the ultimate stresses are 400 MPa in tension, 320 MPa in shear and 640 MPa in crushing, find the minimum force per pitch which will rupture the joint.

If the above joint is subjected to a load such that the factor of safety is 4, find out the actual stresses developed in the plates and the rivets.

**Solution.** Given:
\[ t = 15 \text{ mm}; \quad d = 25 \text{ mm}; \quad p = 75 \text{ mm}; \quad \sigma_{tu} = 400 \text{ MPa} = 400 \text{ N/mm}^2; \quad \tau_u = 320 \text{ MPa} = 320 \text{ N/mm}^2; \quad \sigma_{cu} = 640 \text{ MPa} = 640 \text{ N/mm}^2 \]

**Minimum force per pitch which will rupture the joint**

Since the ultimate stresses are given, therefore we shall find the ultimate values of the resistances of the joint. We know that ultimate tearing resistance of the plate per pitch,
\[ P_{tu} = (p - d) t \times \sigma_{tu} = (75 - 25)15 \times 400 = 300 \, 000 \text{ N} \]

Ultimate shearing resistance of the rivets per pitch,
\[ P_{su} = n \times \frac{\pi}{4} \times d^2 \times \tau_u = 2 \times \frac{\pi}{4} (25)^2 320 = 314200 \text{ N} \quad \ldots (\because n = 2) \]

and ultimate crushing resistance of the rivets per pitch,
\[ P_{cu} = n \times d \times t \times \sigma_{cu} = 2 \times 25 \times 15 \times 640 = 480 \, 000 \text{ N} \]

From above we see that the minimum force per pitch which will rupture the joint is 300 000 N or 300 kN. \textbf{Ans.}

**Actual stresses produced in the plates and rivets**

Since the factor of safety is 4, therefore safe load per pitch length of the joint
\[ = 300 \, 000/4 = 75 \, 000 \text{ N} \]

Let \( \sigma_{ta}, \tau_a \) and \( \sigma_{ca} \) be the actual tearing, shearing and crushing stresses produced with a safe load of 75 000 N in tearing, shearing and crushing.

We know that actual tearing resistance of the plates \( P_{tu} \),
\[ 75 \, 000 = (p - d) t \times \sigma_{tu} = (75 - 25)15 \times 750 = 750 \, \sigma_{tu} \]
\[ \therefore \quad \sigma_{tu} = 75000/750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \textbf{Ans.} \]

Actual shearing resistance of the rivets \( P_{su} \),
\[ 75 \, 000 = n \times \frac{\pi}{4} \times d^2 \times \tau_a = 2 \times \frac{\pi}{4} (25)^2 982 = 982 \, \tau_a \]
\[ \therefore \quad \tau_a = 75000/982 = 76.4 \text{ N/mm}^2 = 76.4 \text{ MPa} \quad \textbf{Ans.} \]

and actual crushing resistance of the rivets \( P_{cu} \),
\[ 75 \, 000 = n \times d \times t \times \sigma_{cu} = 2 \times 25 \times 15 \times 750 \sigma_{cu} \]
\[ \therefore \quad \sigma_{cu} = 75000/750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \textbf{Ans.} \]

**Example 9.2.** Find the efficiency of the following riveted joints:
1. Single riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 50 mm.
2. Double riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 65 mm.

Assume
Permissible tensile stress in plate $= 120$ MPa
Permissible shearing stress in rivets $= 90$ MPa
Permissible crushing stress in rivets $= 180$ MPa

**Solution.** Given: $t = 6$ mm; $d = 20$ mm; $\sigma_t = 120$ MPa $= 120$ N/mm$^2$; $\tau = 90$ MPa $= 90$ N/mm$^2$; $\sigma_c = 180$ MPa $= 180$ N/mm$^2$

1. **Efficiency of the first joint**

   **Pitch,** $p = 50$ mm ...(Given)

   First of all, let us find the tearing resistance of the plate, shearing and crushing resistances of the rivets.

   **(i) Tearing resistance of the plate**

   We know that the tearing resistance of the plate per pitch length,
   $$P_t = (p - d) \times \sigma_t = (50 - 20) \times 12 \times 120 = 21600 \text{ N}$$

   **(ii) Shearing resistance of the rivet**

   Since the joint is a single riveted lap joint, therefore the strength of one rivet in single shear is taken. We know that shearing resistance of one rivet,
   $$P_s = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} (20)^2 \times 90 = 28278 \text{ N}$$

   **(iii) Crushing resistance of the rivet**

   Since the joint is a single riveted, therefore strength of one rivet is taken. We know that crushing resistance of one rivet,
   $$P_c = d \times t \times \sigma_c = 20 \times 6 \times 180 = 21600 \text{ N}$$

   $\therefore$ Strength of the joint
   $$= \text{Least of } P_t, P_s \text{ and } P_c = 21600 \text{ N}$$

   We know that strength of the unriveted or solid plate,
   $$P = p \times t \times \sigma_t = 50 \times 6 \times 120 = 36000 \text{ N}$$

   $\therefore$ Efficiency of the joint,
   $$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{P} = \frac{21600}{36000} = 0.60 \text{ or } 60\% \quad \text{Ans.}$$

2. **Efficiency of the second joint**

   **Pitch,** $p = 65$ mm ...(Given)

   **(i) Tearing resistance of the plate,**

   We know that the tearing resistance of the plate per pitch length,
   $$P_t = (p - d) \times \sigma_t = (65 - 20) \times 6 \times 120 = 32400 \text{ N}$$

   **(ii) Shearing resistance of the rivets**

   Since the joint is double riveted lap joint, therefore strength of two rivets in single shear is taken. We know that shearing resistance of the rivets,
   $$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (20)^2 \times 90 = 56556 \text{ N}$$

   **(iii) Crushing resistance of the rivet**

   Since the joint is double riveted, therefore strength of two rivets is taken. We know that crushing resistance of rivets,
   $$P_c = n \times d \times t \times \sigma_c = 2 \times 20 \times 6 \times 180 = 43200 \text{ N}$$

   $\therefore$ Strength of the joint
   $$= \text{Least of } P_t, P_s \text{ and } P_c = 32400 \text{ N}$$
We know that the strength of the unriveted or solid plate,
\[ P = p \times t \times \sigma_t = 65 \times 6 \times 120 = 46 800 \text{ N} \]
∴ Efficiency of the joint,
\[ \eta = \frac{\text{Least of } P, P_s \text{ and } P_c}{p} = \frac{32 400}{46 800} = 0.692 \text{ or } 69.2\% \text{ Ans.} \]

**Example 9.3.** A double riveted double cover butt joint in plates 20 mm thick is made with 25 mm diameter rivets at 100 mm pitch. The permissible stresses are:
\[ \sigma_t = 120 \text{ MPa}; \quad \tau = 100 \text{ MPa}; \quad \sigma_c = 150 \text{ MPa} \]
Find the efficiency of joint, taking the strength of the rivet in double shear as twice than that of single shear.

**Solution.** Given: \( t = 20 \text{ mm} \); \( d = 25 \text{ mm} \); \( p = 100 \text{ mm} \); \( \sigma_t = 120 \text{ N/mm}^2 \);
\( \tau = 100 \text{ N/mm}^2 \); \( \sigma_c = 150 \text{ N/mm}^2 \)

First of all, let us find the tearing resistance of the plate, shearing resistance and crushing resistance of the rivet.

(i) **Tearing resistance of the plate**

We know that tearing resistance of the plate per pitch length,
\[ P_t = (p - d) \times t \times \sigma_t = (100 	imes 25) 	imes 20 \times 120 = 180 000 \text{ N} \]

(ii) **Shearing resistance of the rivets**

Since the joint is double riveted butt joint, therefore the strength of two rivets in double shear is taken. We know that shearing resistance of the rivets,
\[ P_s = n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 2 \times \frac{\pi}{4} (25)^2 \times 100 = 196 375 \text{ N} \]

(iii) **Crushing resistance of the rivets**

Since the joint is double riveted, therefore the strength of two rivets is taken. We know that crushing resistance of the rivets,
\[ P_c = n \times d \times t \times \sigma_c = 2 \times 25 \times 20 \times 150 = 150 000 \text{ N} \]
∴ Strength of the joint
\[ = \text{Least of } P_s, P_s \text{ and } P_c \]
\[ = 150 000 \text{ N} \]

**Efficiency of the joint**

We know that the strength of the unriveted or solid plate,
\[ P = p \times t \times \sigma_t = 100 \times 20 \times 120 \]
\[ = 240 000 \text{ N} \]
∴ Efficiency of the joint
\[ = \frac{\text{Least of } P, P_s \text{ and } P_c}{P} = \frac{150 000}{240 000} = 0.625 \text{ or } 62.5\% \text{ Ans.} \]

**9.15 Design of Boiler Joints**

The boiler has a longitudinal joint as well as circumferential joint. The **longitudinal joint** is used to join the ends of the plate to get the required diameter of a boiler. For this purpose, a butt joint with two cover plates is used. The
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circumferential joint is used to get the required length of the boiler. For this purpose, a lap joint with one ring overlapping the other alternately is used.

Since a boiler is made up of number of rings, therefore the longitudinal joints are staggered for convenience of connecting rings at places where both longitudinal and circumferential joints occur.

9.16 Assumptions in Designing Boiler Joints

The following assumptions are made while designing a joint for boilers:

1. The load on the joint is equally shared by all the rivets. The assumption implies that the shell and plate are rigid and that all the deformation of the joint takes place in the rivets themselves.
2. The tensile stress is equally distributed over the section of metal between the rivets.
3. The shearing stress in all the rivets is uniform.
4. The crushing stress is uniform.
5. There is no bending stress in the rivets.
6. The holes into which the rivets are driven do not weaken the member.
7. The rivet fills the hole after it is driven.
8. The friction between the surfaces of the plate is neglected.

9.17 Design of Longitudinal Butt Joint for a Boiler

According to Indian Boiler Regulations (I.B.R), the following procedure should be adopted for the design of longitudinal butt joint for a boiler.

1. Thickness of boiler shell. First of all, the thickness of the boiler shell is determined by using the thin cylindrical formula, i.e.

\[ t = \frac{P \cdot D}{2 \cdot \sigma_t \times \eta_l} + 1 \text{ mm as corrosion allowance} \]

where
- \( t \) = Thickness of the boiler shell,
- \( P \) = Steam pressure in boiler,
- \( D \) = Internal diameter of boiler shell,
- \( \sigma_t \) = Permissible tensile stress, and
- \( \eta_l \) = Efficiency of the longitudinal joint.

The following points may be noted:

(a) The thickness of the boiler shell should not be less than 7 mm.
(b) The efficiency of the joint may be taken from the following table.

<table>
<thead>
<tr>
<th>Lap joints</th>
<th>Efficiency (%)</th>
<th>*Maximum efficiency</th>
<th>Butt joints (Double strap)</th>
<th>Efficiency (%)</th>
<th>*Maximum efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single riveted</td>
<td>45 to 60</td>
<td>63.3</td>
<td>Single riveted</td>
<td>55 to 60</td>
<td>63.3</td>
</tr>
<tr>
<td>Double riveted</td>
<td>63 to 70</td>
<td>77.5</td>
<td>Double riveted</td>
<td>70 to 83</td>
<td>86.6</td>
</tr>
<tr>
<td>Triple riveted</td>
<td>72 to 80</td>
<td>86.6</td>
<td>Triple riveted (5 rivets per pitch with unequal width of straps)</td>
<td>80 to 90</td>
<td>95.0</td>
</tr>
<tr>
<td>Quadruple riveted</td>
<td>85 to 94</td>
<td>98.1</td>
<td>Quadruple riveted</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The maximum efficiencies are valid for ideal equistrength joints with tensile stress = 77 MPa, shear stress = 62 MPa and crushing stress = 133 MPa.
Indian Boiler Regulations (I.B.R.) allow a maximum efficiency of 85% for the best joint. According to I.B.R., the factor of safety should not be less than 4. The following table shows the values of factor of safety for various kind of joints in boilers.

**Table 9.2. Factor of safety for boiler joints.**

<table>
<thead>
<tr>
<th>Type of joint</th>
<th>Factor of safety</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hand riveting</td>
</tr>
<tr>
<td>Lap joint</td>
<td>4.75</td>
</tr>
<tr>
<td>Single strap butt joint</td>
<td>4.75</td>
</tr>
<tr>
<td>Single riveted butt joint with two equal cover straps</td>
<td>4.75</td>
</tr>
<tr>
<td>Double riveted butt joint with two equal cover straps</td>
<td>4.25</td>
</tr>
</tbody>
</table>

2. **Diameter of rivets.** After finding out the thickness of the boiler shell (t), the diameter of the rivet hole (d) may be determined by using Unwin’s empirical formula, *i.e.*

\[
d = 6 \sqrt{t}
\]

(when \( t \) is greater than 8 mm)

But if the thickness of plate is less than 8 mm, then the diameter of the rivet hole may be calculated by equating the shearing resistance of the rivets to crushing resistance. In no case, the diameter of rivet hole should not be less than the thickness of the plate, because there will be danger of punch crushing. The following table gives the rivet diameter corresponding to the diameter of rivet hole as per IS : 1928 – 1961 (Reaffirmed 1996).

**Table 9.3. Size of rivet diameters for rivet hole diameter as per IS : 1928 – 1961 (Reaffirmed 1996).**

<table>
<thead>
<tr>
<th>Basic size of rivet mm</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
<th>22</th>
<th>24</th>
<th>27</th>
<th>30</th>
<th>33</th>
<th>36</th>
<th>39</th>
<th>42</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rivet hole diameter (min) mm</td>
<td>13</td>
<td>15</td>
<td>17</td>
<td>19</td>
<td>21</td>
<td>23</td>
<td>25</td>
<td>28.5</td>
<td>31.5</td>
<td>34.5</td>
<td>37.5</td>
<td>41</td>
<td>44</td>
<td>50</td>
</tr>
</tbody>
</table>

According to IS : 1928 – 1961 (Reaffirmed 1996), the table on the next page (Table 9.4) gives the preferred length and diameter combination for rivets.

3. **Pitch of rivets.** The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. It may noted that

(a) The pitch of the rivets should not be less than 2d, which is necessary for the formation of head.

(b) The maximum value of the pitch of rivets for a longitudinal joint of a boiler as per I.B.R. is

\[
p_{\text{max}} = C \times t + 41.28 \text{ mm}
\]

where

\[
t = \text{Thickness of the shell plate in mm, and}
\]

\[
C = \text{Constant.}
\]

The value of the constant C is given in Table 9.5.
### Table 9.4. Preferred length and diameter combinations for rivets used in boilers as per IS: 1928–1961 (Reaffirmed 1996).

(All dimensions in mm)

<table>
<thead>
<tr>
<th>Length</th>
<th>Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>28</td>
<td>×</td>
</tr>
<tr>
<td>31.5</td>
<td>× ×</td>
</tr>
<tr>
<td>35.5</td>
<td>× × ×</td>
</tr>
<tr>
<td>40</td>
<td>× × × ×</td>
</tr>
<tr>
<td>45</td>
<td>× × × × ×</td>
</tr>
<tr>
<td>50</td>
<td>× × × × × ×</td>
</tr>
<tr>
<td>56</td>
<td>× × × × × × ×</td>
</tr>
<tr>
<td>63</td>
<td>× × × × × × × ×</td>
</tr>
<tr>
<td>71</td>
<td>× × × × × × × × ×</td>
</tr>
<tr>
<td>80</td>
<td>× × × × × × × × ×</td>
</tr>
<tr>
<td>85</td>
<td>– × × × × × × × × ×</td>
</tr>
<tr>
<td>90</td>
<td>– × × × × × × × × × ×</td>
</tr>
<tr>
<td>95</td>
<td>– × × × × × × × × × × ×</td>
</tr>
<tr>
<td>100</td>
<td>– – × × × × × × × × × × ×</td>
</tr>
<tr>
<td>106</td>
<td>– – – × × × × × × × × × × ×</td>
</tr>
<tr>
<td>112</td>
<td>– – – – × × × × × × × × × × ×</td>
</tr>
<tr>
<td>118</td>
<td>– – – – – × × × × × × × × × × ×</td>
</tr>
<tr>
<td>125</td>
<td>– – – – – – × × × × × × × × × × ×</td>
</tr>
<tr>
<td>132</td>
<td>– – – – – – – × × × × × × × × × × ×</td>
</tr>
<tr>
<td>140</td>
<td>– – – – – – – – × × × × × × × × × × ×</td>
</tr>
<tr>
<td>150</td>
<td>– – – – – – – – – × × × × × × × × × × ×</td>
</tr>
<tr>
<td>160</td>
<td>– – – – – – – – – – × × × × × × × × × × ×</td>
</tr>
<tr>
<td>180</td>
<td>– – – – – – – – – – – × × × × × × × × × × ×</td>
</tr>
<tr>
<td>200</td>
<td>– – – – – – – – – – – – × × × × × × × × × × ×</td>
</tr>
<tr>
<td>224</td>
<td>– – – – – – – – – – – – – × × × × × × × × × × ×</td>
</tr>
<tr>
<td>250</td>
<td>– – – – – – – – – – – – – – × × × × × × × × × × ×</td>
</tr>
</tbody>
</table>

Preferred numbers are indicated by ×.

### Table 9.5. Values of constant C.

<table>
<thead>
<tr>
<th>Number of rivets per pitch length</th>
<th>Lap joint</th>
<th>Butt joint (single strap)</th>
<th>Butt joint (double strap)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.31</td>
<td>1.53</td>
<td>1.75</td>
</tr>
<tr>
<td>2</td>
<td>2.62</td>
<td>3.06</td>
<td>3.50</td>
</tr>
<tr>
<td>3</td>
<td>3.47</td>
<td>4.05</td>
<td>4.63</td>
</tr>
<tr>
<td>4</td>
<td>4.17</td>
<td>–</td>
<td>5.52</td>
</tr>
<tr>
<td>5</td>
<td>–</td>
<td>–</td>
<td>6.00</td>
</tr>
</tbody>
</table>
Note: If the pitch of rivets as obtained by equating the tearing resistance to the shearing resistance is more than $p_{\text{max}}$, then the value of $p_{\text{max}}$ is taken.

4. **Distance between the rows of rivets.** The distance between the rows of rivets as specified by Indian Boiler Regulations is as follows:

   (a) For equal number of rivets in more than one row for lap joint or butt joint, the distance between the rows of rivets ($p_b$) should not be less than $0.33p + 0.67d$, for zig-zig riveting, and $2d$, for chain riveting.

   (b) For joints in which the number of rivets in outer rows is half the number of rivets in inner rows and if the inner rows are chain riveted, the distance between the outer rows and the next rows should not be less than $0.33p + 0.67d$ or $2d$, whichever is greater.

   The distance between the rows in which there are full number of rivets shall not be less than $2d$.

   (c) For joints in which the number of rivets in outer rows is half the number of rivets in inner rows and if the inner rows are zig-zig riveted, the distance between the outer rows and the next rows shall not be less than $0.2p + 1.15d$. The distance between the rows in which there are full number of rivets (zig-zag) shall not be less than $0.165p + 0.67d$.

Note: In the above discussion, $p$ is the pitch of the rivets in the outer rows.

5. **Thickness of butt strap.** According to I.B.R., the thicknesses for butt strap ($t_1$) are as given below:

   (a) The thickness of butt strap, in no case, shall be less than 10 mm.

   (b) $t_1 = 1.125t$, for ordinary (chain riveting) single butt strap.

   $t_1 = 1.125t \left( \frac{p - d}{p - 2d} \right)$, for single butt straps, every alternate rivet in outer rows being omitted.

   $t_1 = 0.625t$, for double butt-straps of equal width having ordinary riveting (chain riveting).

   $t_1 = 0.625t \left( \frac{p - d}{p - 2d} \right)$, for double butt straps of equal width having every alternate rivet in the outer rows being omitted.

   (c) For unequal width of butt straps, the thicknesses of butt strap are $t_1 = 0.75t$, for wide strap on the inside, and $t_3 = 0.625t$, for narrow strap on the outside.

6. **Margin.** The margin ($m$) is taken as $1.5d$.

Note: The above procedure may also be applied to ordinary riveted joints.

9.18 **Design of Circumferential Lap Joint for a Boiler**

The following procedure is adopted for the design of circumferential lap joint for a boiler.

1. **Thickness of the shell and diameter of rivets.** The thickness of the boiler shell and the diameter of the rivet will be the same as for longitudinal joint.

2. **Number of rivets.** Since it is a lap joint, therefore the rivets will be in single shear.

   $\therefore$ Shearing resistance of the rivets,

   $$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau$$  \hspace{1cm} \text{...(i)}
where \( n = \) Total number of rivets.

Knowing the inner diameter of the boiler shell \((D)\), and the pressure of steam \((P)\), the total shearing load acting on the circumferential joint,

\[
W_s = \frac{\pi}{4} \times D^2 \times P
\]

...(ii)

From equations (i) and (ii), we get

\[
n \times \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times D^2 \times P
\]

\[
\therefore \quad n = \left( \frac{D}{d} \right)^2 \frac{P}{\tau}
\]

Fig. 9.18. Longitudinal and circumferential joint.

3. Pitch of rivets. If the efficiency of the longitudinal joint is known, then the efficiency of the circumferential joint may be obtained. It is generally taken as 50\% of tearing efficiency in longitudinal joint, but if more than one circumferential joints is used, then it is 62\% for the intermediate joints. Knowing the efficiency of the circumferential lap joint \((\eta_c)\), the pitch of the rivets for the lap joint
(p₁) may be obtained by using the relation:

\[ \eta_c = \frac{p_t - d}{p_t} \]

4. Number of rows. The number of rows of rivets for the circumferential joint may be obtained from the following relation:

Number of rows = \[ \frac{\text{Total number of rivets}}{\text{Number of rivets in one row}} \]

and the number of rivets in one row

\[ = \frac{\pi (D + t)}{p_t} \]

where \( D \) = Inner diameter of shell.

5. After finding out the number of rows, the type of the joint (i.e. single riveted or double riveted etc.) may be decided. Then the number of rivets in a row and pitch may be re-adjusted. In order to have a leak-proof joint, the pitch for the joint should be checked from Indian Boiler Regulations.

6. The distance between the rows of rivets (i.e. back pitch) is calculated by using the relations as discussed in the previous article.

7. After knowing the distance between the rows of rivets \( p_b \), the overlap of the plate may be fixed by using the relation,

Overlap = \( (\text{No. of rows of rivets} - 1) \cdot p_b + m \)

where \( m \) = Margin.

There are several ways of joining the longitudinal joint and the circumferential joint. One of the methods of joining the longitudinal and circumferential joint is shown in Fig. 9.18.

**9.19 Recommended Joints for Pressure Vessels**

The following table shows the recommended joints for pressure vessels.

### Table 9.6. Recommended joints for pressure vessels.

<table>
<thead>
<tr>
<th>Diameter of shell (metres)</th>
<th>Thickness of shell (mm)</th>
<th>Type of joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 to 1.8</td>
<td>6 to 13</td>
<td>Double riveted</td>
</tr>
<tr>
<td>0.9 to 2.1</td>
<td>13 to 25</td>
<td>Triple riveted</td>
</tr>
<tr>
<td>1.5 to 2.7</td>
<td>19 to 40</td>
<td>Quadruple riveted</td>
</tr>
</tbody>
</table>

**Example 9.4.** A double riveted lap joint with zig-zag riveting is to be designed for 13 mm thick plates. Assume

\[ \sigma_t = 80 \text{ MPa} ; \tau = 60 \text{ MPa} ; \text{ and } \sigma_c = 120 \text{ MPa} \]

State how the joint will fail and find the efficiency of the joint.

**Solution.** Given : \( t = 13 \text{ mm} ; \sigma_t = 80 \text{ MPa} = 80 \text{ N/mm}^2 ; \tau = 60 \text{ MPa} = 60 \text{ N/mm}^2 ; \sigma_c = 120 \text{ MPa} = 120 \text{ N/mm}^2 \)

1. Diameter of rivet

Since the thickness of plate is greater than 8 mm, therefore diameter of rivet hole,

\[ d = 6\sqrt{t} = 6\sqrt{13} = 21.6 \text{ mm} \]

From Table 9.3, we find that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard size of the rivet hole \( (d) \) is 23 mm and the corresponding diameter of the rivet is 22 mm. **Ans.**
2. **Pitch of rivets**

Let \( p \) = Pitch of the rivets.

Since the joint is a double riveted lap joint with zig-zag riveting [See Fig. 9.6 (c)], therefore there are two rivets per pitch length, \( i.e. n = 2 \). Also, in a lap joint, the rivets are in single shear.

We know that tearing resistance of the plate,

\[
P_t = (p - d) t \times \sigma_t = (p - 23) 13 \times 80 = (p - 23) 1040 \text{ N} \quad \ldots (i)
\]

and shearing resistance of the rivets,

\[
P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (23)^2 60 = 49864 \text{ N} \quad \ldots (ii)
\]

From equations (i) and (ii), we get

\[
p - 23 = \frac{49864}{1040} = 48 \quad \text{or} \quad p = 48 + 23 = 71 \text{ mm}
\]

The maximum pitch is given by,

\[
p_{\text{max}} = C \times t + 41.28 \text{ mm}
\]

From Table 9.5, we find that for 2 rivets per pitch length, the value of \( C \) is 2.62.

\[
\therefore \quad p_{\text{max}} = 2.62 \times 13 + 41.28 = 75.28 \text{ mm}
\]

Since \( p_{\text{max}} \) is more than \( p \), therefore we shall adopt

\[
p = 71 \text{ mm} \quad \text{Ans.}
\]

3. **Distance between the rows of rivets**

We know that the distance between the rows of rivets (for zig-zag riveting),

\[
p_b = 0.33 p + 0.67 d = 0.33 \times 71 + 0.67 \times 23 \text{ mm}
\]

\[
= 38.8 \text{ say 40 mm} \quad \text{Ans.}
\]

4. **Margin**

We know that the margin,

\[
m = 1.5 d = 1.5 \times 23 = 34.5 \text{ say 35 mm} \quad \text{Ans.}
\]

**Failure of the joint**

Now let us find the tearing resistance of the plate, shearing resistance and crushing resistance of the rivets.

We know that tearing resistance of the plate,

\[
P_t = (p - d) t \times \sigma_t = (71 - 23)13 \times 80 = 49920 \text{ N}
\]

Shearing resistance of the rivets,

\[
P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (23)^2 60 = 49864 \text{ N}
\]

and crushing resistance of the rivets,

\[
P_c = n \times d \times t \times \sigma_c = 2 \times 23 \times 13 \times 120 = 71760 \text{ N}
\]

The least of \( P_t, P_s \) and \( P_c \) is \( P_t = 49864 \text{ N} \). Hence the joint will fail due to shearing of the rivets. **Ans.**

**Efficiency of the joint**

We know that strength of the un riveted or solid plate,

\[
P = p \times t \times \sigma_t = 71 \times 13 \times 80 = 73840 \text{ N}
\]

\[
\therefore \quad \text{Efficiency of the joint}, \quad \eta = \frac{P_t}{P} = \frac{49864}{73840} = 0.675 \text{ or } 67.5\% \quad \text{Ans.}
\]
Example 9.5. Two plates of 7 mm thick are connected by a triple riveted lap joint of zig-zag pattern. Calculate the rivet diameter, rivet pitch and distance between rows of rivets for the joint. Also state the mode of failure of the joint. The safe working stresses are as follows:

\[ \sigma_t = 90 \text{ MPa} ; \quad \tau = 60 \text{ MPa} ; \quad \text{and} \quad \sigma_c = 120 \text{ MPa}. \]

Solution. Given: \( t = 7 \text{ mm} \); \( \sigma_t = 90 \text{ MPa} = 90 \text{ N/mm}^2 \); \( \tau = 60 \text{ MPa} = 60 \text{ N/mm}^2 \); \( \sigma_c = 120 \text{ MPa} = 120 \text{ N/mm}^2 \)

1. Diameter of rivet

Since the thickness of plate is less than 8 mm, therefore diameter of the rivet hole (\( d \)) is obtained by equating the shearing resistance (\( P_s \)) to the crushing resistance (\( P_c \)) of the rivets. The triple riveted lap joint of zig-zag pattern is shown in Fig. 9.7(b). We see that there are three rivets per pitch length (i.e. \( n = 3 \)). Also, the rivets in lap joint are in single shear.

We know that shearing resistance of the rivets,

\[
P_s = n \times \frac{\pi}{4} \times d^2 \times \tau
\]

\[= 3 \times \frac{\pi}{4} \times d^2 \times 60 = 141.4 \times d^2 \text{ N} \quad \text{...(i)}
\]

and crushing resistance of the rivets,

\[
P_c = n \times d \times t \times \sigma_c = 3 \times d \times 7 \times 120 = 2520 \times d \text{ N} \quad \text{...(ii)}
\]

From equations (i) and (ii), we get

\[141.4 \times d^2 = 2520 \times d \quad \text{or} \quad d = \frac{2520}{141.4} = 17.8 \text{ mm}
\]

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of rivet hole (\( d \)) is 19 mm and the corresponding diameter of rivet is 18 mm. Answ.

2. Pitch of rivets

Let \( p = \) Pitch of rivets.

We know that tearing resistance of the plate,

\[
P_t = (p - d) \times t \times \sigma_t = (p - 19) \times 7 \times 90 = 630 \times (p - 19) \text{ N} \quad \text{...(iii)}
\]

and shearing resistance of the rivets,

\[
P_s = 141.4 \times d^2 = 141.4 \times (19)^2 = 51,045 \text{ N} \quad \text{...(iv)}
\]

Equating equations (iii) and (iv), we get

\[630 \times (p - 19) = 51,045 \]

\[p - 19 = \frac{51,045}{630} = 81 \quad \text{or} \quad p = 81 + 19 = 100 \text{ mm}
\]
3. Distance between rows of rivets

We know that the distance between the rows of rivets for zig-zag riveting,

\[ p_b = 0.33p + 0.67d = 0.33 \times 66 + 0.67 \times 19 = 34.5 \text{ mm} \quad \text{Ans.} \]

Mode of failure of the joint

We know that tearing resistance of the plate,

\[ P_t = (p - d) \times \sigma_t = (66 - 19) \times 90 = 29610 \text{ N} \]

Shearing resistance of rivets,

\[ P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 3 \times \frac{\pi}{4} (19)^2 60 = 51045 \text{ N} \]

and crushing resistance of rivets.

\[ P_c = n \times d \times t \times \sigma_c = 3 \times 19 \times 7 \times 120 = 47880 \text{ N} \]

From above we see that the least value of \( P_t, P_s \) and \( P_c \) is \( P_t = 29610 \text{ N} \). Therefore the joint will fail due to tearing off the plate.

Example 9.6. Two plates of 10 mm thickness each are to be joined by means of a single riveted double strap butt joint. Determine the rivet diameter, rivet pitch, strap thickness and efficiency of the joint. Take the working stresses in tension and shearing as 80 MPa and 60 MPa respectively.

Solution. Given : \( \tau = 10 \text{ mm} ; \sigma_t = 80 \text{ MPa} = 80 \text{ N/mm}^2 ; \tau = 60 \text{ MPa} = 60 \text{ N/mm}^2 \)

1. Diameter of rivet

Since the thickness of plate is greater than 8 mm, therefore diameter of rivet hole,

\[ d = 6 \sqrt{t} = 6 \sqrt{10} = 18.97 \text{ mm} \]

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of rivet hole \( d \) is 19 mm and the corresponding diameter of the rivet is 18 mm. \( \text{Ans.} \)

2. Pitch of rivets

Let \( p = \) Pitch of rivets.

Since the joint is a single riveted double strap butt joint as shown in Fig. 9.8, therefore there is one rivet per pitch length \( (i.e. n = 1) \) and the rivets are in double shear.

We know that tearing resistance of the plate,

\[ P_t = (p - d) \times \sigma_t = (p - 19) \times 10 \times 80 = 800 (p - 19) \text{ N} \quad \text{...(i)} \]

and shearing resistance of the rivets,

\[ P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{...(\because Rivets are in double shear)}\]

\[ = 1 \times 1.875 \times \frac{\pi}{4} (19)^2 60 = 31900 \text{ N} \quad \text{...(\because n = 1)} \quad \text{...(ii)} \]

From equations (i) and (ii), we get

\[ 800 (p - 19) = 31900 \]

\[ \therefore p - 19 = 31900 / 800 = 39.87 \text{ or } p = 39.87 + 19 = 58.87 \text{ say 60 mm} \]

According to I.B.R., the maximum pitch of rivets,

\[ P_{max} = C \times t + 41.28 \text{ mm} \]
From Table 9.5, we find that for double strap butt joint and 1 rivet per pitch length, the value of $C$ is 1.75.
\[ p_{\text{max}} = 1.75 \times 10 = 17.5 \text{ say } 18 \text{ mm} \]
From above we see that $p = p_{\text{max}} = 60 \text{ mm}$ \textbf{Ans.}

3. **Thickness of cover plates**
   We know that thickness of cover plates,
   
   
   \[ t_1 = 0.625 \times t = 0.625 \times 10 = 6.25 \text{ mm} \] \textbf{Ans.}

**Efficiency of the joint**

We know that tearing resistance of the plate,

\[ P_t = (p - d) \times \sigma_t = (60 - 19) \times 80 = 32800 \text{ N} \]

and shearing resistance of the rivets,

\[ P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 1 \times \frac{\pi}{4} \times 19^2 = 31900 \text{ N} \]

\[ \therefore \text{ Strength of the joint } = \text{ Least of } P_t \text{ and } P_s = 31900 \text{ N} \]

Strength of the unriveted plate per pitch length

\[ P = p \times t \times \sigma_t = 60 \times 10 \times 80 = 48000 \text{ N} \]

\[ \therefore \text{ Efficiency of the joint, } \eta = \frac{\text{Least of } P \text{ and } P_s}{P} = \frac{31900}{48000} = 0.665 \text{ or } 66.5\% \] \textbf{Ans.}

**Example 9.7.** Design a double riveted butt joint with two cover plates for the longitudinal seam of a boiler shell 1.5 m in diameter subjected to a steam pressure of 0.95 N/mm$^2$. Assume joint efficiency as 75%, allowable tensile stress in the plate 90 MPa; compressive stress 140 MPa; and shear stress in the rivet 56 MPa.

**Solution.** Given:

\[ D = 1.5 \text{ m} = 1500 \text{ mm} ; \]
\[ P = 0.95 \text{ N/mm}^2 ; \]
\[ \eta_l = 75\% = 0.75 ; \]
\[ \sigma_t = 90 \text{ MPa} = 90 \text{ N/mm}^2 ; \]
\[ \sigma_c = 140 \text{ MPa} = 140 \text{ N/mm}^2 ; \]
\[ \tau = 56 \text{ MPa} = 56 \text{ N/mm}^2 \]

1. **Thickness of boiler shell plate**
   We know that thickness of boiler shell plate,

   \[ t = \frac{P.D}{2 \sigma_t \times \eta_l} + 1 \text{ mm} = \frac{0.95 \times 1500}{2 \times 90 \times 0.75} + 1 = 11.6 \text{ say } 12 \text{ mm} \] \textbf{Ans.}

2. **Diameter of rivet**
   Since the thickness of the plate is greater than 8 mm, therefore the diameter of the rivet hole,

   \[ d = 6\sqrt{t} = 6\sqrt{12} = 20.8 \text{ mm} \]

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of the rivet hole ($d$) is 21 mm and the corresponding diameter of the rivet is 20 mm. \textbf{Ans.}

3. **Pitch of rivets**
   Let
   \[ p = \text{Pitch of rivets.} \]

   The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

   We know that tearing resistance of the plate,

   \[ P_t = (p - d) \times \sigma_t = (p - 21) \times 90 \times 1080 = (p - 21) \text{N} \quad \ldots(i) \]

   Since the joint is double riveted double strap butt joint, as shown in Fig. 9.9, therefore there are two rivets per pitch length (i.e. $n = 2$) and the rivets are in double shear. Assuming that the rivets in
double shear are 1.875 times stronger than in single shear, we have

\[ P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 1.875 \times \frac{\pi}{4} (21)^2 \times 56 \text{ N} \]

\[ = 72,745 \text{ N} \]  

\( \text{...(ii)} \)

From equations (i) and (ii), we get

\[ 1080 (p - 21) = 72,745 \]

\[ \therefore p - 21 = 72,745 / 1080 = 67.35 \text{ or } p = 67.35 + 21 = 88.35 \text{ say 90 mm} \]

According to I.B.R., the maximum pitch of rivets for longitudinal joint of a boiler is given by

\[ p_{\text{max}} = C \times t + 41.28 \text{ mm} \]

From Table 9.5, we find that for a double riveted double strap butt joint and two rivets per pitch length, the value of \( C \) is 3.50.

\[ \therefore p_{\text{max}} = 3.5 \times 12 + 41.28 = 83.28 \text{ say 84 mm} \]

Since the value of \( p \) is more than \( p_{\text{max}} \), therefore we shall adopt pitch of the rivets,

\[ p = p_{\text{max}} = 84 \text{ mm} \]

\( \text{Ans.} \)

4. **Distance between rows of rivets**

Assuming zig-zag riveting, the distance between the rows of the rivets (according to I.B.R.),

\[ p_b = 0.33p + 0.67d = 0.33 \times 84 + 0.67 \times 21 = 41.8 \text{ say 42 mm} \]

\( \text{Ans.} \)

5. **Thickness of cover plates**

According to I.B.R., the thickness of each cover plate of equal width is

\[ t_1 = 0.625 \times t = 0.625 \times 12 = 7.5 \text{ mm} \]

\( \text{Ans.} \)

6. **Margin**

We know that the margin,

\[ m = 1.5d = 1.5 \times 21 = 31.5 \text{ say 32 mm} \]

\( \text{Ans.} \)

Let us now find the efficiency for the designed joint.

Tearing resistance of the plate,

\[ P_t = (p - d) \times t \times \sigma_t = (84 - 21)12 \times 90 = 68040 \text{ N} \]

Shearing resistance of the rivets,

\[ P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 1.875 \times \frac{\pi}{4} (21)^2 \times 56 = 72,745 \text{ N} \]

and crushing resistance of the rivets,

\[ P_c = n \times d \times t \times \sigma_c = 2 \times 21 \times 12 \times 140 = 70,560 \text{ N} \]

Since the strength of riveted joint is the least value of \( P_t, P_s \) or \( P_c \), therefore strength of the riveted joint,

\[ P = 68040 \text{ N} \]

We know that strength of the un-riveted plate,

\[ P = p \times t \times \sigma_t = 84 \times 12 \times 90 = 90,720 \text{ N} \]

\[ \therefore \text{Efficiency of the designed joint,} \]

\[ \eta = \frac{P}{P} = \frac{68040}{90,720} = 0.75 \text{ or 75%} \]

\( \text{Ans.} \)

Since the efficiency of the designed joint is equal to the given efficiency of 75%, therefore the design is satisfactory.
Example 9.8. A pressure vessel has an internal diameter of 1 m and is to be subjected to an internal pressure of 2.75 N/mm² above the atmospheric pressure. Considering it as a thin cylinder and assuming efficiency of its riveted joint to be 79%, calculate the plate thickness if the tensile stress in the material is not to exceed 88 MPa.

Design a longitudinal double riveted double strap butt joint with equal straps for this vessel. The pitch of the rivets in the outer row is to be double the pitch in the inner row and zig-zag riveting is proposed. The maximum allowable shear stress in the rivets is 64 MPa. You may assume that the rivets in double shear are 1.8 times stronger than in single shear and the joint does not fail by crushing.

Make a sketch of the joint showing all calculated values. Calculate the efficiency of the joint.

Solution. Given: \( D = 1 \text{ m} = 1000 \text{ mm} \); \( P = 2.75 \text{ N/mm}^2 \); \( \eta_l = 79\% = 0.79 \); \( \sigma_t = 88 \text{ MPa} = 88 \text{ N/mm}^2 \); \( \tau = 64 \text{ MPa} = 64 \text{ N/mm}^2 \)

1. Thickness of plate

We know that the thickness of plate,

\[
t = \frac{P \cdot D}{2 \cdot \sigma_t \times \eta_l} + 1 \text{ mm} = \frac{2.75 \times 1000}{2 \times 88 \times 0.79} + 1 \text{ mm}
\]

\[= 20.8 \text{ say } 21 \text{ mm} \quad \text{Ans.}
\]

2. Diameter of rivet

Since the thickness of plate is more than 8 mm, therefore diameter of rivet hole,

\[d = 6 \sqrt{t} = 6 \sqrt{21} = 27.5 \text{ mm}
\]

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of the rivet hole \( (d) \) is 28.5 mm and the corresponding diameter of the rivet is 27 mm. \textbf{Ans.}

3. Pitch of rivets

Let \( p \) = Pitch in the outer row.

The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

We know that the tearing resistance of the plate per pitch length,

\[
P_t = (p - d) \times \sigma_t = (p - 28.5) \times 21 \times 88 = 1848 \times (p - 28.5) \text{ N} \quad \text{...(i)}
\]

Since the pitch in the outer row is twice the pitch of the inner row and the joint is double riveted, therefore for one pitch length there will be three rivets in double shear \( (i.e. \ n = 3) \). It is given that the strength of rivets in double shear is 1.8 times that of single shear, therefore

Shearing strength of the rivets per pitch length,

\[
P_s = n \times 1.8 \times \frac{\pi}{4} \times d^2 \times \tau = 3 \times 1.8 \times \frac{\pi}{4} (28.5)^2 \times 64 \text{ N}
\]

\[= 220 \ 500 \text{ N} \quad \text{...(ii)}
\]

From equations \((i)\) and \((ii)\), we get

\[1848 \times (p - 28.5) = 220 \ 500 \]

\[
p - 28.5 = 220 \ 500 / 1848 \Rightarrow 119.3
\]

or

\[p = 119.3 + 28.5 = 147.8 \text{ mm}
\]
According to I.B.R., the maximum pitch, 
\[ p_{\text{max}} = C \times t + 41.28 \text{ mm} \]

From Table 9.5, we find that for 3 rivets per pitch length and for double strap butt joint, the value of \( C \) is 4.63. 
\[ p_{\text{max}} = 4.63 \times 21 + 41.28 = 138.5 \text{ say 140 mm} \]
Since the value of \( p_{\text{max}} \) is less than \( p \), therefore we shall adopt the value of 
\[ p = p_{\text{max}} = 140 \text{ mm} \quad \text{Ans.} \]
∴ Pitch in the inner row 
\[ = 140 / 2 = 70 \text{ mm} \quad \text{Ans.} \]

4. Distance between the rows of rivets 
According to I.B.R., the distance between the rows of rivets, 
\[ p_b = 0.2 p + 1.15 d = 0.2 \times 140 + 1.15 \times 28.5 = 61 \text{ mm} \quad \text{Ans.} \]

5. Thickness of butt strap 
According to I.B.R., the thickness of double butt straps of equal width, 
\[ t_1 = 0.625 \left( \frac{p - d}{p - 2d} \right) = 0.625 \times 21 \left( \frac{140 - 28.5}{140 - 2 \times 28.5} \right) \text{ mm} \]
\[ = 17.6 \text{ say 18 mm} \quad \text{Ans.} \]

6. Margin 
We know that the margin, 
\[ m = 1.5 d = 1.5 \times 28.5 = 43 \text{ mm} \quad \text{Ans.} \]

Efficiency of the joint 
We know that tearing resistance of the plate, 
\[ P_t = (p - d) t \times \sigma_t = (140 - 28.5) \times 21 \times 88 = 206,050 \text{ N} \]
Shearing resistance of the rivets, 
\[ P_s = n \times 1.8 \times \pi \times \frac{d}{4} \times d^2 \times \tau = 3 \times 1.8 \times \pi \times \frac{28.5}{4} (28.5)^2 64 = 220,500 \text{ N} \]
Strength of the solid plate, 
\[ = p \times t \times \sigma_t = 140 \times 21 \times 88 = 258,720 \text{ N} \]
∴ Efficiency of the joint 
\[ = \frac{\text{Least of } P_t \text{ and } P_s}{\text{Strength of solid plate}} = \frac{206,050}{258,720} = 0.796 \text{ or 79.6\%} \quad \text{Ans.} \]

Since the efficiency of the designed joint is more than the given efficiency, therefore the design is satisfactory.

Example 9.9. Design the longitudinal joint for a 1.25 m diameter steam boiler to carry a steam pressure of 2.5 N/mm². The ultimate strength of the boiler plate may be assumed as 420 MPa, crushing strength as 650 MPa and shear strength as 300 MPa. Take the joint efficiency as 80\%. Sketch the joint with all the dimensions. Adopt the suitable factor of safety.

Solution. Given: \( D = 1.25 \text{ m} = 1250 \text{ mm}; P = 2.5 \text{ N/mm}^2; \sigma_{tu} = 420 \text{ MPa} = 420 \text{ N/mm}^2; \sigma_{cu} = 650 \text{ MPa} = 650 \text{ N/mm}^2; \tau_u = 300 \text{ MPa} = 300 \text{ N/mm}^2; \eta = 80\% = 0.8 \)

Assuming a factor of safety (F.S.) as 5, the allowable stresses are as follows:
\[ \sigma_t = \frac{\sigma_{tu}}{F.S.} = \frac{420}{5} = 84 \text{ N/mm}^2 \]
\[ \sigma_c = \frac{\sigma_{cu}}{F.S.} = \frac{650}{5} = 130 \text{ N/mm}^2 \]
and \[ \tau = \frac{\tau_{w}}{F.S.} = \frac{300}{5} = 60 \text{ N/mm}^2 \]

1. **Thickness of plate**
   
   We know that thickness of plate, 
   
   \[ t = \frac{P.D}{2 \sigma_{t} \times \eta_{t}} + 1 \text{ mm} = \frac{2.5 \times 1250}{2 \times 84 \times 0.8} + 1 \text{ mm} \]
   
   \[ = 24.3 \text{ say } 25 \text{ mm} \quad \text{Ans.} \]

2. **Diameter of rivet**

   Since the thickness of the plate is more than 8 mm, therefore diameter of the rivet hole,
   
   \[ d = 6 \sqrt{t} = 6 \sqrt{25} = 30 \text{ mm} \]

   From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of the rivet hole is 31.5 mm and the corresponding diameter of the rivet is 30 mm.  \text{An}.

3. **Pitch of rivets**

   Assume a triple riveted double strap butt joint with unequal straps, as shown in Fig. 9.11.

   Let \[ p = \text{Pitch of the rivets in the outer most row}. \]

   \[ \therefore \text{Tearing strength of the plate per pitch length}, \]

   \[ P_{t} = (p - d) \times \sigma_{t} = (p - 31.5) \times 25 = 2100 \times (p - 31.5) \text{ N} \quad \text{(i)} \]

   Since the joint is triple riveted with two unequal cover straps, therefore there are 5 rivets per pitch length. Out of these five rivets, four rivets are in double shear and one is in single shear.

   Assuming the strength of the rivets in double shear as 1.875 times that of single shear, therefore

   Shearing resistance of the rivets per pitch length,

   \[ P_{s} = 4 \times 1.875 \times \frac{\pi}{4} \times d^{2} \times \tau + \frac{\pi}{4} \times d^{2} \times \tau = 8.5 \times \frac{\pi}{4} \times d^{2} \times \tau \]

   \[ = 8.5 \times \frac{\pi}{4} (31.5)^{2} 60 = 397500 \text{ N} \quad \text{...(ii)} \]

   From equations (i) and (ii), we get

   \[ 2100 (p - 31.5) = 397500 \]

   \[ \therefore \]

   \[ p - 31.5 = 189.3 \text{ or } p = 31.5 + 189.3 = 220.8 \text{ mm} \]

   According to I.B.R., maximum pitch,

   \[ P_{\text{max}} = C \times t + 41.28 \text{ mm} \]

   From Table 9.5, we find that for double strap butt joint with 5 rivets per pitch length, the value of \( C \) is 6.

   \[ \therefore \]

   \[ P_{\text{max}} = 6 \times 25 + 41.28 = 191.28 \text{ say } 196 \text{ mm} \quad \text{Ans.} \]

   Since \( P_{\text{max}} \) is less than \( p \), therefore we shall adopt \( p = P_{\text{max}} = 196 \text{ mm} \quad \text{Ans.} \]

   \[ \therefore \text{Pitch of rivets in the inner row}, \]

   \[ p' = 196 / 2 = 98 \text{ mm} \quad \text{Ans.} \]

4. **Distance between the rows of rivets**

   According to I.B.R., the distance between the outer row and the next row,

   \[ = 0.2 p + 1.15 d = 0.2 \times 196 + 1.15 \times 31.5 \text{ mm} \]

   \[ = 75.4 \text{ say } 76 \text{ mm} \quad \text{Ans.} \]

   and the distance between the inner rows for zig-zag riveting

   \[ = 0.165 p + 0.67 d = 0.165 \times 196 + 0.67 \times 31.5 \text{ mm} \]

   \[ = 53.4 \text{ say } 54 \text{ mm} \quad \text{Ans.} \]
5. **Thickness of butt straps**

We know that for unequal width of butt straps, the thicknesses are as follows:

For wide butt strap, \( t_1 = 0.75 \times 25 = 18.75 \) say 20 mm \textbf{Ans.} \\
and for narrow butt strap, \( t_2 = 0.625 \times 25 = 15.6 \) say 16 mm \textbf{Ans.}

It may be noted that wide and narrow butt straps are placed on the inside and outside of the shell respectively.

6. **Margin**

We know that the margin, \( m = 1.5 \times 31.5 = 47.25 \) say 47.5 mm \textbf{Ans.}

Let us now check the efficiency of the designed joint.

Tearing resistance of the plate in the outer row, 
\[ P_t = (p - d) \times t \times \sigma_t = (196 - 31.5) \times 25 \times 84 = 345450 \text{ N} \]

Shearing resistance of the rivets, 
\[ P_s = 4 \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau + \frac{\pi}{4} \times d^2 \times \tau = 8.5 \times \frac{\pi}{4} \times d^2 \times \tau \]
\[ = 8.5 \times \frac{\pi}{4} (31.5)^2 \times 60 = 397500 \text{ N} \]

and crushing resistance of the rivets, 
\[ P_c = n \times d \times t \times \sigma_c = 5 \times 31.5 \times 25 \times 130 = 511875 \text{ N} \]
\[ \cdots (\because n = 5) \]

The joint may also fail by tearing off the plate between the rivets in the second row. This is only possible if the rivets in the outermost row gives way (i.e. shears). Since there are two rivet holes per pitch length in the second row and one rivet is in the outermost row, therefore combined tearing and shearing resistance

\[ = (p - 2d) \times t \times \sigma_t + \frac{\pi}{4} \times d^2 \times \tau \]
\[ = (196 - 2 \times 31.5) \times 25 \times 84 + \frac{\pi}{4} (31.5)^2 \times 60 = 326065 \text{ N} \]

From above, we see that strength of the joint
\[ = 326065 \text{ N} \]

Strength of the unriveted or solid plate, 
\[ P = p \times t \times \sigma_t = 196 \times 25 \times 84 = 411600 \text{ N} \]
\[ \therefore \text{ Efficiency of the joint,} \]
\[ \eta = 326065 / 411600 = 0.792 \text{ or } 79.2\% \]

Since the efficiency of the designed joint is nearly equal to the given efficiency, therefore the design is satisfactory.

**Example 9.10.** A steam boiler is to be designed for a working pressure of 2.5 N/mm\(^2\) with its inside diameter 1.6 m. Give the design calculations for the longitudinal and circumferential joints for the following working stresses for steel plates and rivets:

\[ \text{In tension} = 75 \text{ MPa; In shear} = 60 \text{ MPa; In crushing} = 125 \text{ MPa.} \]

Draw the joints to a suitable scale.

**Solution.** Given : \( P = 2.5 \text{ N/mm}^2; \ D = 1.6 \text{ m} = 1600 \text{ mm}; \sigma_t = 75 \text{ MPa} = 75 \text{ N/mm}^2; \)
\[ \tau = 60 \text{ MPa} = 60 \text{ N/mm}^2; \sigma_c = 125 \text{ MPa} = 125 \text{ N/mm}^2 \]

**Design of longitudinal joint**

The longitudinal joint for a steam boiler may be designed as follows:
1. **Thickness of boiler shell**

We know that the thickness of boiler shell,

\[ t = \frac{P \cdot D}{2 \sigma_t} + 1 \text{ mm} = \frac{2.5 \times 1600}{2 \times 75} + 1 \text{ mm} = 27.6 \text{ say 28 mm} \quad \text{Ans.} \]

2. **Diameter of rivet**

Since the thickness of the plate is more than 8 mm, therefore diameter of rivet hole,

\[ d = 6 \sqrt[3]{t} = 6 \sqrt[3]{28} = 31.75 \text{ mm} \]

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of rivet hole \((d)\) is 34.5 mm and the corresponding diameter of the rivet is 33 mm. \quad \text{Ans.}

3. **Pitch of rivets**

Assume the joint to be triple riveted double strap butt joint with unequal cover straps, as shown in Fig. 9.11.

Let \( p = \) Pitch of the rivet in the outer most row.

\[ \therefore \text{Tearing resistance of the plate per pitch length,} \]

\[ P_t = (p - d) t \times \sigma_t = (p - 34.5) 28 \times 75 \text{ N} \]

\[ = 2100 (p - 34.5) \text{ N} \quad \ldots(i) \]

Since the joint is triple riveted with two unequal cover straps, therefore there are 5 rivets per pitch length. Out of these five rivets, four are in double shear and one is in single shear. Assuming the strength of rivets in double shear as 1.875 times that of single shear, therefore

Shearing resistance of the rivets per pitch length,

\[ P_s = 4 \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau + \frac{\pi}{4} \times d^2 \times \tau \]

\[ = 8.5 \times \frac{\pi}{4} \times d^2 \times \tau \]

\[ = 8.5 \times \frac{\pi}{4} \times (34.5)^2 \times 60 = 476 \text{ 820 N} \quad \ldots(ii) \]

Equating equations \((i)\) and \((ii)\), we get

\[ 2100 (p - 34.5) = 476 \text{ 820} \]

\[ \therefore p - 34.5 = 476 \text{ 820} / 2100 = 227 \text{ or } p = 227 + 34.5 = 261.5 \text{ mm} \]

According to I.B.R., the maximum pitch,

\[ p_{\text{max}} = C.t + 41.28 \text{ mm} \]

From Table 9.5, we find that for double strap butt joint with 5 rivets per pitch length, the value of \(C\) is 6.

\[ \therefore p_{\text{max}} = 6 \times 28 + 41.28 \text{ say 220 mm} \]

Since \(p_{\text{max}}\) is less than \(p\), therefore we shall adopt

\[ p = p_{\text{max}} = 220 \text{ mm} \quad \text{Ans.} \]

\[ \therefore \text{Pitch of rivets in the inner row,} \]

\[ p' = 220 / 2 = 110 \text{ mm} \quad \text{Ans.} \]

4. **Distance between the rows of rivets**

According to I.B.R., the distance between the outer row and the next row

\[ = 0.2 p + 1.15 d = 0.2 \times 220 + 1.15 \times 34.5 \text{ mm} \]

\[ = 83.7 \text{ say 85 mm} \quad \text{Ans.} \]
and the distance between the inner rows for zig-zig riveting
\[ \frac{1}{2} (p + 0.67 d) = 0.165 \times 220 + 0.67 \times 34.5 \text{ mm} \]
\[ = 59.4 \text{ say } 60 \text{ mm} \quad \text{Ans.} \]

5. Thickness of butt straps

We know that for unequal width of butt straps, the thicknesses are:

For wide butt strap,
\[ t_1 = 0.75 \times 28 = 21 \text{ mm} \quad \text{Ans.} \]
and for narrow butt strap,
\[ t_2 = 0.625 \times 28 = 17.5 \text{ say } 18 \text{ mm} \quad \text{Ans.} \]

It may be noted that the wide and narrow butt straps are placed on the inside and outside of the shell respectively.

6. Margin

We know that the margin,
\[ m = 1.5 d = 1.5 \times 34.5 = 51.75 \text{ say } 52 \text{ mm} \quad \text{Ans.} \]

Let us now check the efficiency of the designed joint.

Tearing resistance of the plate in the outer row,
\[ P_t = (p - d) t \times \sigma_t = (220 - 34.5) 28 \times 75 = 389,550 \text{ N} \]

Shearing resistance of the rivets,
\[ P_s = 4 \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau + \frac{\pi}{4} \times d^2 \times \tau = 8.5 \times \frac{\pi}{4} \times d^2 \times \tau \]
\[ = 8.5 \times \frac{\pi}{4} (34.5)^2 60 = 476,820 \text{ N} \]

and crushing resistance of the rivets,
\[ P_c = n \times d \times t \times \sigma_c = 5 \times 34.5 \times 28 \times 125 = 603,750 \text{ N} \]

The joint may also fail by tearing off the plate between the rivets in the second row. This is only possible if the rivets in the outermost row give way (i.e., shears). Since there are two rivet holes per pitch length in the second row and one rivet in the outermost row, therefore

Combined tearing and shearing resistance
\[ = (p - 2d) t \times \sigma_t + \frac{\pi}{4} \times d^2 \times \tau \]
\[ = (220 - 2 \times 34.5) 28 \times 75 + \frac{\pi}{4} (34.5)^2 60 \]
\[ = 317,100 + 56,096 = 376,196 \text{ N} \]

From above, we see that the strength of the joint
\[ = 373,196 \text{ N} \]

Strength of the unriveted or solid plate,
\[ P = p \times t \times \sigma_t = 220 \times 28 \times 75 = 462,000 \text{ N} \]

\[ \therefore \text{ Efficiency of the designed joint,} \]
\[ \eta = \frac{373,196}{462,000} = 0.808 \text{ or } 80.8\% \quad \text{Ans.} \]

Design of circumferential joint

The circumferential joint for a steam boiler may be designed as follows:

1. The thickness of the boiler shell \( t \) and diameter of rivet hole \( d \) will be same as for longitudinal joint, i.e.
\[ t = 28 \text{ mm} ; \text{ and } d = 34.5 \text{ mm} \]
2. **Number of rivets**

Let \( n \) = Number of rivets.

We know that shearing resistance of the rivets

\[
= n \times \frac{\pi}{4} \times d^2 \times \tau
\]

...\( (i) \)

and total shearing load acting on the circumferential joint

\[
= \frac{\pi}{4} \times D^2 \times P
\]

...\( (ii) \)

From equations \( (i) \) and \( (ii) \), we get

\[
\frac{n \times \frac{\pi}{4} \times d^2 \times \tau}{\frac{\pi}{4} \times D^2 \times P} \Rightarrow n = \frac{D^2 \times P}{d^2 \times \tau} = \frac{(1600)^2 \times 2.5}{(34.5)^2 \times 60} = 89.6 \text{ say 90} \quad \text{Ans.}
\]

3. **Pitch of rivets**

Assuming the joint to be double riveted lap joint with zig-zag riveting, therefore number of rivets per row

\[
= \frac{90}{2} = 45
\]

We know that the pitch of the rivets,

\[
p_1 = \frac{\pi (D + t)}{\text{Number of rivets per row}} = \frac{\pi (1600 + 28)}{45} = 113.7 \text{ mm}
\]

Let us take pitch of the rivets, \( p_1 = 140 \text{ mm} \quad \text{Ans.} \)

4. **Efficiency of the joint**

We know that the efficiency of the circumferential joint,

\[
\eta_c = \frac{p_1 - d}{p_1} = \frac{140 - 34.5}{140} = 0.753 \text{ or } 75.3\%
\]

5. **Distance between the rows of rivets**

We know that the distance between the rows of rivets for zig-zag riveting,

\[
= 0.33 p_1 + 0.67 d = 0.33 \times 140 + 0.67 \times 34.5 \text{ mm}
\]

\[
= 69.3 \text{ say 70 mm} \quad \text{Ans.}
\]

6. **Margin**

We know that the margin,

\[
m = 1.5 d = 1.5 \times 34.5
\]

\[
= 51.75 \text{ say 52 mm} \quad \text{Ans.}
\]

### 9.20 Riveted Joint for Structural Use—Joints of Uniform Strength (Lozenge Joint)

A riveted joint known as **Lozenge joint** used for roof, bridge work or girders etc. is shown in Fig. 9.19. In such a joint, diamond riveting is employed so that the joint is made of uniform strength.

*Fig. 9.19 shows a triple riveted double strap butt joint.*

Riveted joints are used for roofs, bridge work and girders.
Let $b =$ Width of the plate, 
$t =$ Thickness of the plate, and 
$d =$ Diameter of the rivet hole.

In designing a Lozenge joint, the following procedure is adopted.

1. **Diameter of rivet**

The diameter of the rivet hole is obtained by using Unwin’s formula, *i.e.*

$$d = 6 \sqrt{t}$$

According to IS : 1929–1982 (Reaffirmed 1996), the sizes of rivets for general purposes are given in the following table.

<table>
<thead>
<tr>
<th>Diameter of rivet hole (mm)</th>
<th>13.5</th>
<th>15.5</th>
<th>17.5</th>
<th>19.5</th>
<th>21.5</th>
<th>23.5</th>
<th>25.5</th>
<th>29</th>
<th>32</th>
<th>35</th>
<th>38</th>
<th>41</th>
<th>44</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of rivet (mm)</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
<td>24</td>
<td>27</td>
<td>30</td>
<td>33</td>
<td>36</td>
<td>39</td>
<td>42</td>
<td>48</td>
</tr>
</tbody>
</table>

2. **Number of rivets**

The number of rivets required for the joint may be obtained by the shearing or crushing resistance of the rivets.

Let

$$P_t = \text{Maximum pull acting on the joint. This is the tearing resistance of the plate at the outer row which has only one rivet.}$$

$$= (b - d) t \times \sigma_t$$

and

$$n = \text{Number of rivets.}$$

Since the joint is double strap butt joint, therefore the rivets are in double shear. It is assumed that resistance of a rivet in double shear is 1.75 times than in single shear in order to allow for possible eccentricity of load and defective workmanship.
Shearing resistance of one rivet,
\[ P_s = 1.75 \times \frac{\pi}{4} \times d^2 \times \tau \]
and crushing resistance of one rivet,
\[ P_c = d \times t \times \sigma_c \]

\[ \therefore \text{Number of rivets required for the joint,} \]
\[ n = \frac{P_t}{\text{Least of } P_r \text{ or } P_c} \]

3. From the number of rivets, the number of rows and the number of rivets in each row is decided.

4. **Thickness of the butt straps**

The thickness of the butt strap,
\[ t_1 = 1.25 \, t, \text{ for single cover strap} \]
\[ = 0.75 \, t, \text{ for double cover strap} \]

5. **Efficiency of the joint**

First of all, calculate the resistances along the sections 1-1, 2-2 and 3-3.

At section 1-1, there is only one rivet hole.

\[ \therefore \text{Resistance of the joint in tearing along 1-1,} \]
\[ P_{t1} = (b - d) \times t \times \sigma_t \]

At section 2-2, there are two rivet holes.

\[ \therefore \text{Resistance of the joint in tearing along 2-2,} \]
\[ P_{t2} = (b - 2d) \times t \times \sigma_t + \text{Strength of one rivet in front of section 2-2} \]

(This is due to the fact that for tearing off the plate at section 2-2, the rivet in front of section 2-2 i.e. at section 1-1 must first fracture).

Similarly at section 3-3 there are three rivet holes.

\[ \therefore \text{Resistance of the joint in tearing along 3-3,} \]
\[ P_{t3} = (b - 3d) \times t \times \sigma_t + \text{Strength of 3 rivets in front of section 3-3} \]

The least value of \( P_{t1}, P_{t2}, P_{t3}, P_s \) or \( P_c \) is the strength of the joint.

We know that the strength of unriveted plate,
\[ P = b \times t \times \sigma_t \]

\[ \therefore \text{Efficiency of the joint,} \]
\[ \eta = \frac{\text{Least of } P_{t1}, P_{t2}, P_{t3}, P_s \text{ or } P_c}{P} \]

**Note:** The permissible stresses employed in structural joints are higher than those used in design of pressure vessels. The following values are usually adopted.

- For plates in tension ... 140 MPa
- For rivets in shear ... 105 MPa
- For crushing of rivets and plates
  - Single shear ... 224 MPa
  - Double shear ... 280 MPa

6. The pitch of the rivets is obtained by equating the strength of the joint in tension to the strength of the rivets in shear. The pitches allowed in structural joints are larger than those of pressure vessels. The following table shows the values of pitch due to Rotscher.
7. The marginal pitch \((m)\) should not be less than \(1.5 \, d\).
8. The distance between the rows of rivets is \(2.5 \, d\) to \(3 \, d\).

**Example 9.11.** Two lengths of mild steel tie rod having width 200 mm and thickness 12.5 mm are to be connected by means of a butt joint with double cover plates. Design the joint if the permissible stresses are 80 MPa in tension, 65 MPa in shear and 160 MPa in crushing. Make a sketch of the joint.

**Solution.** Given: \(b = 200 \, \text{mm} \); \(t = 12.5 \, \text{mm} \); \(\sigma_t = 80 \, \text{MPa} = 80 \, \text{N/mm}^2 \); \(\tau = 65 \, \text{MPa} = 65 \, \text{N/mm}^2 \); \(\sigma_c = 160 \, \text{MPa} = 160 \, \text{N/mm}^2 \)

1. **Diameter of rivet**

   We know that the diameter of rivet hole,
   \[
d = 6 \sqrt[6]{t} = 6 \sqrt[6]{12.5} = 21.2 \, \text{mm}
   \]

   From Table 9.7, we see that according to IS : 1929 – 1982 (Reaffirmed 1996), the standard diameter of the rivet hole \((d)\) is 21.5 mm and the corresponding diameter of rivet is 20 mm. **Ans.**

2. **Number of rivets**

   Let \(n = \text{Number of rivets}\).

   We know that maximum pull acting on the joint,
   \[
P_t = (b - d) \times t \times \sigma_t = (200 - 21.5) \times 12.5 \times 80 = 178 500 \, \text{N}
   \]

   Since the joint is a butt joint with double cover plates as shown in Fig. 9.20, therefore the rivets are in double shear. Assume that the resistance of the rivet in double shear is 1.75 times than in single shear.

   \[
   \therefore \text{Shearing resistance of one rivet},
   \]

   \[
P_s = 1.75 \times \frac{\pi}{4} \times d^2 \times \tau = 1.75 \times \frac{\pi}{4} \times (21.5)^2 \times 65 = 41 300 \, \text{N}
   \]

   and crushing resistance of one rivet,

   \[
P_c = d \times t \times \sigma_c = 21.5 \times 12.5 \times 160 = 43 000 \, \text{N}
   \]

   Since the shearing resistance is less than the crushing resistance, therefore number of rivets required for the joint,

   \[
n = \frac{P_t}{P_s} = \frac{178 500}{41 300} = 4.32 \, \text{say} \, 5 \, \text{Ans.}
   \]

---

**Table 9.8. Pitch of rivets for structural joints.**

<table>
<thead>
<tr>
<th>Thickness of plate (mm)</th>
<th>Diameter of rivet hole (mm)</th>
<th>Diameter of rivet (mm)</th>
<th>Pitch of rivet (p = 3d + 5)mm</th>
<th>Marginal pitch (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8.4</td>
<td>8</td>
<td>29</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>9.5</td>
<td>9</td>
<td>32</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>10</td>
<td>35</td>
<td>17</td>
</tr>
<tr>
<td>5–6</td>
<td>13</td>
<td>12</td>
<td>38</td>
<td>18</td>
</tr>
<tr>
<td>6–8</td>
<td>15</td>
<td>14</td>
<td>47</td>
<td>21</td>
</tr>
<tr>
<td>8–12</td>
<td>17</td>
<td>16</td>
<td>56</td>
<td>25</td>
</tr>
<tr>
<td>11–15</td>
<td>21</td>
<td>20</td>
<td>65</td>
<td>30</td>
</tr>
</tbody>
</table>
3. The arrangement of the rivets is shown in Fig. 9.20.

4. **Thickness of butt straps**
   
   We know that thickness of butt straps,
   
   $$ t_1 = 0.75 \times 12.5 = 9.375 \text{ say } 9.4 \text{ mm} \quad \text{Ans.} $$

5. **Efficiency of the joint**

   First of all, let us find the resistances along the sections 1-1, 2-2 and 3-3.

   At section 1-1, there is only one rivet hole.
   
   ∴ Resistance of the joint in tearing along section 1-1,
   
   $$ P_{t1} = (b - d) t \times \sigma_t = (200 - 21.5) 12.5 \times 80 = 178500 \text{ N} $$

   At section 2-2, there are two rivet holes. In this case, the tearing of the plate will only take place if the rivet at section 1-1 (in front of section 2-2) gives way (i.e. shears).
   
   ∴ Resistance of the joint in tearing along section 2-2,
   
   $$ P_{t2} = (b - 2d) t \times \sigma_t + \text{Shearing resistance of one rivet} \\
   = (200 - 2 \times 21.5) 12.5 \times 80 + 41300 = 198300 \text{ N} $$

   At section 3-3, there are two rivet holes. The tearing of the plate will only take place if one rivet at section 1-1 and two rivets at section 2-2 gives way (i.e. shears).
   
   ∴ Resistance of the joint in tearing along section 3-3,
   
   $$ P_{t3} = (b - 2d) t \times \sigma_t + \text{Shearing resistance of 3 rivets} \\
   = (200 - 2 \times 21.5) 12.5 \times 80 + 3 \times 41300 = 280900 \text{ N} $$

   Shearing resistance of all the 5 rivets
   
   $$ P_s = 5 \times 41300 = 206500 \text{ N} $$

   and crushing resistance of all the 5 rivets,
   
   $$ P_c = 5 \times 43000 = 215000 \text{ N} $$
Since the strength of the joint is the least value of $P_1$, $P_2$, $P_3$, $P_s$ and $P_c$, therefore strength of the joint

$$= 178500 \text{ N along section 1-1}$$

We know that strength of the un-riveted plate,

$$= b \times t \times \sigma_t = 20 \times 12.5 \times 80 = 200000 \text{ N}$$

\[ \therefore \text{ Efficiency of the joint,} \]

$$\eta = \frac{\text{Strength of the joint}}{\text{Strength of the unriveted plate}} = \frac{178500}{200000} = 0.8925 \text{ or } 89.25\% \quad \text{Ans.} \]

6. Pitch of rivets,

$$p = 3d + 5 \text{ mm} = (3 \times 21.5) + 5 = 69.5 \text{ say 70 mm} \quad \text{Ans.}$$

7. Marginal pitch,

$$m = 1.5d = 1.5 \times 21.5 = 33.25 \text{ say 35 mm} \quad \text{Ans.}$$

8. Distance between the rows of rivets

$$= 2.5d = 2.5 \times 21.5 = 53.75 \text{ say 55 mm} \quad \text{Ans.}$$

**Example 9.12.** A tie-bar in a bridge consists of flat 350 mm wide and 20 mm thick. It is connected to a gusset plate of the same thickness by a double cover butt joint. Design an economical joint if the permissible stresses are:

$$\sigma_t = 90 \text{ MPa}, \tau = 60 \text{ MPa and } \sigma_c = 150 \text{ MPa}$$

**Solution.** Given: $b = 350 \text{ mm} ; t = 20 \text{ mm} ; \sigma_t = 90 \text{ MPa} = 90 \text{ N/mm}^2 ; \tau = 60 \text{ MPa} = 60 \text{ N/mm}^2 ; \sigma_c = 150 \text{ MPa} = 150 \text{ N/mm}^2$

Riveted, screwed and welded joints are employed in bridges.
1. **Diameter of rivet**

   We know that the diameter of rivet hole,
   \[ d = 6 \sqrt{r} = 6 \sqrt{20} = 26.8 \text{ mm} \]

   From Table 9.7, we see that according to IS : 1929–1982 (Reaffirmed 1996), the standard diameter of rivet hole \((d)\) is 29 mm and the corresponding diameter of rivet is 27 mm. **Ans.**

2. **Number of rivets**

   Let \( n \) = Number of rivets.

   We know that the maximum pull acting on the joint,
   \[ P_t = (b - d) t \times \sigma_t = (350 - 29) 20 \times 90 = 577 800 \text{ N} \]

   Since the joint is double strap butt joint, therefore the rivets are in double shear. Assume that the resistance of the rivet in double shear is 1.75 times than in single shear.

   .

   Shearing resistance of one rivet,
   \[ P_s = 1.75 \times \frac{\pi}{4} \times d^2 \times \frac{\tau}{\pi} = 1.75 \times \frac{\pi}{4} (29)^2 60 = 69 360 \text{ N} \]

   and crushing resistance of one rivet,
   \[ P_c = d \times t \times \sigma_c = 29 \times 20 \times 150 = 87 000 \text{ N} \]

   Since the shearing resistance is less than crushing resistance, therefore number of rivets required for the joint,
   \[ n = \frac{P_t}{P_s} = \frac{577 800}{69 360} = 8.33 \text{ say } 9 \text{ Ans.} \]

3. The arrangement of rivets is shown in Fig. 9.21.

   ![Fig. 9.21](image_url)

   **Fig. 9.21.** All dimensions in mm.

4. **Thickness of butt straps**

   We know that the thickness of butt straps,
   \[ t_1 = 0.75 \times 20 = 15 \text{ mm Ans.} \]
5. Efficiency of the joint

First of all, let us find the resistances along the sections 1-1, 2-2, 3-3 and 4-4.

At section 1-1, there is only one rivet hole.
∴ Resistance of the joint in tearing along 1-1,

\[ P_{t1} = (b - d) t \times \sigma_t = (350 - 29) 20 \times 90 = 577,800 \text{ N} \]

At section 2-2, there are two rivet holes. In this case the tearing of the plate will only take place if the rivet at section 1-1 (in front of section 2-2) gives way.
∴ Resistance of the joint in tearing along 2-2,

\[ P_{t2} = (b - 2d) t \times \sigma_t + \text{Shearing strength of one rivet in front} \\
= (350 - 2 \times 29) 20 \times 90 + 69,360 = 594,960 \text{ N} \]

At section 3-3, there are three rivet holes. The tearing of the plate will only take place if one rivet at section 1-1 and two rivets at section 2-2 gives way.
∴ Resistance of the joint in tearing along 3-3,

\[ P_{t3} = (b - 3d) t \times \sigma_t + \text{Shearing strength of 3 rivets in front} \\
= (350 - 3 \times 29) 20 \times 90 + 3 \times 69,360 = 681,480 \text{ N} \]

Similarly, resistance of the joint in tearing along 4-4,

\[ P_{t4} = (b - 3d) t \times \sigma_t + \text{Shearing strength of 6 rivets in front} \\
= (350 - 3 \times 29) 20 \times 90 + 6 \times 69,360 = 889,560 \text{ N} \]

Shearing resistance of all the 9 rivets,
\[ P_s = 9 \times 69,360 = 624,240 \text{ N} \]

and crushing resistance of all the 9 rivets,
\[ P_c = 9 \times 87,000 = 783,000 \text{ N} \]

The strength of the joint is the least of \(P_{t1}, P_{t2}, P_{t3}, P_{t4}, P_s\) and \(P_c\).
∴ Strength of the joint
\[ = 577,800 \text{ N} \text{ along section 1-1} \]

We know that the strength of the un-riveted plate,
\[ P = b \times t \times \sigma_t = 350 \times 20 \times 90 = 630,000 \text{ N} \]
∴ Efficiency of the joint,
\[ \eta = \frac{\text{Strength of the joint}}{\text{Strength of the un-riveted plate}} = \frac{577,800}{630,000} = 0.917 \text{ or } 91.7\% \text{ Ans.} \]

6. Pitch of rivets,
\[ p = 3d + 5 \text{ mm} = 3 \times 29 + 5 = 92 \text{ say 95 mm} \text{ Ans.} \]

7. Marginal pitch,
\[ m = 1.5d = 1.5 \times 29 = 43.5 \text{ say 45 mm} \text{ Ans.} \]

8. Distance between the rows of rivets
\[ = 2.5d = 2.5 \times 29 = 72.5 \text{ say 75 mm} \text{ Ans.} \]

Note: If chain riveting with three rows of three rivets in each is used instead of diamond riveting, then

Least strength of the joint
\[ = (b - 3d) t \times \sigma_t = (350 - 3 \times 29) 20 \times 90 = 473,400 \text{ N} \]
∴ Efficiency of the joint
\[ = \frac{473,400}{630,000} = 0.752 \text{ or } 75.2\% \]

Thus we see that with the use of diamond riveting, efficiency of the joint is increased.
Example 9.13. Design a lap joint for a mild steel flat tie-bar 200 mm × 10 mm thick, using 24 mm diameter rivets. Assume allowable stresses in tension and compression of the plate material as 112 MPa and 200 MPa respectively and shear stress of the rivets as 84 MPa. Show the disposition of the rivets for maximum joint efficiency and determine the joint efficiency. Take diameter of rivet hole as 25.5 mm for a 24 mm diameter rivet.

Solution. Given: \( b = 200 \text{ mm} \); \( t = 10 \text{ mm} \); \( \sigma_t = 112 \text{ MPa} = 112 \text{ N/mm}^2 \); \( \sigma_c = 200 \text{ MPa} = 200 \text{ N/mm}^2 \); \( \tau = 84 \text{ MPa} = 84 \text{ N/mm}^2 \); \( d = 25.5 \text{ mm} \); \( d_1 = 24 \text{ mm} \)

1. Number of rivets

Let \( n = \) Number of rivets.

We know that the maximum pull acting on the joint,

\[
P_t = (b - d) t \times \sigma_t = (200 - 25.5) 10 \times 112 = 195 440 \text{ N}
\]

Since the joint is a lap joint, therefore shearing resistance of one rivet,

\[
P_s = \frac{\pi}{4} d^2 \times \tau = \frac{\pi}{4} (25.5)^2 84 = 42 905 \text{ N}
\]

and crushing resistance of one rivet,

\[
P_c = d \times t \times \sigma_c = 25.5 \times 10 \times 200 = 51 000 \text{ N}
\]

Since the shearing resistance is less than the crushing resistance, therefore number of rivets required for the joint,

\[
n = \frac{P_t}{P_s} = \frac{195 440}{42 905} = 4.56 \text{ say 5} \quad \text{Ans.}
\]

2. The arrangement of the rivets is shown in Fig. 9.22.

3. Thickness of the cover plate

We know that the thickness of a cover plate for lap joint,

\[
t_1 = 1.25 t = 1.25 \times 10 = 12.5 \text{ mm} \quad \text{Ans.}
\]

4. Efficiency of the joint

First of all, let us find the resistances along the sections 1-1, 2-2 and 3-3. At section 1-1, there is only one rivet hole.

\[
\therefore \text{Resistance of the joint in tearing along section 1-1,} \]

\[
P_{\text{t1}} = (b - d) t \times \sigma_t = (200 - 25.5) 10 \times 112 = 195 440 \text{ N}
\]

At section 2-2, there are three rivet holes. In this case, the tearing of the plate will only take place if the rivet at section 1-1 (in front of section 2-2) gives way (i.e. shears).
Resistance of the joint in tearing along section 2-2,

\[ P_{t2} = (b - 3d) t \times \sigma_t + \text{Shearing resistance of one rivet} \]
\[ = (200 - 3 \times 25.5) 10 \times 112 + 42,905 = 181,285 \text{ N} \]

At section 3-3, there is only one rivet hole. The resistance of the joint in tearing along section 3-3 will be same as at section 1-1.

\[ P_{t3} = P_{t1} = 195,440 \text{ N} \]

Shearing resistance of all the five rivets,

\[ P_s = 5 \times 42,905 = 214,525 \text{ N} \]

and crushing resistance of all the five rivets,

\[ P_c = 5 \times 51,000 = 525,000 \text{ N} \]

Since the strength of the joint is the least value of \( P_{t1}, P_{t2}, P_{t3}, P_s, \) and \( P_c \), therefore strength of the joint

\[ = 181,285 \text{ N at section 2-2} \]

We know that strength of the un-riveted plate

\[ = b \times t \times \sigma_t = 200 \times 10 \times 112 = 224,000 \text{ N} \]

\[ \therefore \text{Efficiency of the joint}, \]
\[ \eta = \frac{\text{Strength of the joint}}{\text{Strength of the un-riveted plate}} = \frac{181,225}{224,000} \]
\[ = 0.809 \text{ or } 80.9\% \text{ Ans.} \]

### 9.21 Eccentric Loaded Riveted Joint

When the line of action of the load does not pass through the centroid of the rivet system and thus all rivets are not equally loaded, then the joint is said to be an eccentric loaded riveted joint, as shown in Fig. 9.23 (a). The eccentric loading results in secondary shear caused by the tendency of force to twist the joint about the centre of gravity in addition to direct shear or primary shear.

Let

\[ P = \text{Eccentric load on the joint, and} \]
\[ e = \text{Eccentricity of the load i.e. the distance between the line of action of the load and the centroid of the rivet system i.e. } G. \]

The following procedure is adopted for the design of an eccentrically loaded riveted joint.

**Note:** This picture is given as additional information and is not a direct example of the current chapter.
1. First of all, find the centre of gravity $G$ of the rivet system.

Let $A = \text{Cross-sectional area of each rivet},$

$x_1, x_2, x_3 \text{ etc.} = \text{Distances of rivets from } OY, \text{ and}$

$y_1, y_2, y_3 \text{ etc.} = \text{Distances of rivets from } OX.$

We know that

$$x = \frac{A(2x_1 + x_2 + x_3 + \ldots)}{nA} = \frac{x_1 + x_2 + x_3 + \ldots}{n} \quad \text{(where } n = \text{Number of rivets)$$

Similarly,

$$y = \frac{y_1 + y_2 + y_3 + \ldots}{n}$$

2. Introduce two forces $P_1$ and $P_2$ at the centre of gravity ‘$G$’ of the rivet system. These forces are equal and opposite to $P$ as shown in Fig. 9.23 (b).

3. Assuming that all the rivets are of the same size, the effect of $P_1 = P$ is to produce direct shear load on each rivet of equal magnitude. Therefore, direct shear load on each rivet,

$$P_s = \frac{P}{n}, \text{ acting parallel to the load } P.$$
4. The effect of \( P_2 = P \) is to produce a turning moment of magnitude \( P \times e \) which tends to rotate the joint about the centre of gravity ‘\( G \)’ of the rivet system in a clockwise direction. Due to the turning moment, secondary shear load on each rivet is produced. In order to find the secondary shear load, the following two assumptions are made:

\[(a)\] The secondary shear load is proportional to the radial distance of the rivet under consideration from the centre of gravity of the rivet system.

\[(b)\] The direction of secondary shear load is perpendicular to the line joining the centre of the rivet to the centre of gravity of the rivet system.

Let \( F_1, F_2, F_3, \ldots \) = Secondary shear loads on the rivets 1, 2, 3...etc.

\( l_1, l_2, l_3, \ldots \) = Radial distance of the rivets 1, 2, 3...etc. from the centre of gravity ‘\( G \)’ of the rivet system.

\[\therefore\] From assumption \((a)\),

\[ F_1 \propto l_1 ; F_2 \propto l_2 \text{ and so on} \]

or

\[ \frac{F_1}{l_1} = \frac{F_2}{l_2} = \frac{F_3}{l_3} = \ldots \]

\[\therefore\] \[ F_2 = F_1 \times \frac{l_2}{l_1}, \text{ and } F_3 = F_1 \times \frac{l_3}{l_1} \]

We know that the sum of the external turning moment due to the eccentric load and of internal resisting moment of the rivets must be equal to zero.

\[\therefore\] \[ P_e = F_1 l_1 + F_2 l_2 + F_3 l_3 + \ldots \]

\[= F_1 l_1 + F_1 \times \frac{l_2}{l_1} \times l_2 + F_1 \times \frac{l_3}{l_1} \times l_3 + \ldots \]

\[= \frac{F_1}{l_1} \left[ (l_1)^2 + (l_2)^2 + (l_3)^2 + \ldots \right] \]

From the above expression, the value of \( F_1 \) may be calculated and hence \( F_2 \) and \( F_3 \) etc. are known. The direction of these forces are at right angles to the lines joining the centre of rivet to the centre of gravity of the rivet system, as shown in Fig. 9.23 (b), and should produce the moment in the same direction (i.e. clockwise or anticlockwise) about the centre of gravity, as the turning moment \((P \times e)\).

5. The primary (or direct) and secondary shear load may be added vectorially to determine the resultant shear load \((R)\) on each rivet as shown in Fig. 9.23 (c). It may also be obtained by using the relation

\[ R = \sqrt{(P_1)^2 + F^2 + 2 P_1 F \times \cos \theta} \]

where

\[ \theta = \text{Angle between the primary or direct shear load (} P_1 \) and secondary shear load (} F \).\]

When the secondary shear load on each rivet is equal, then the heavily loaded rivet will be one in which the included angle between the direct shear load and secondary shear load is minimum. The maximum loaded rivet becomes the critical one for determining the strength of the riveted joint. Knowing the permissible shear stress \((\tau)\), the diameter of the rivet hole may be obtained by using the relation,

Maximum resultant shear load \((R)\) = \( \frac{\pi}{4} \times d^2 \times \tau \)

From Table 9.7, the standard diameter of the rivet hole \((d)\) and the rivet diameter may be specified, according to IS : 1929 – 1982 (Reaffirmed 1996).
Notes: 1. In the solution of a problem, the primary and shear loads may be laid off approximately to scale and generally the rivet having the maximum resultant shear load will be apparent by inspection. The values of the load for that rivet may then be calculated.

2. When the thickness of the plate is given, then the diameter of the rivet hole may be checked against crushing.

3. When the eccentric load $P$ is inclined at some angle, then the same procedure as discussed above may be followed to find the size of rivet (See Example 9.18).

Example 9.14. An eccentrically loaded lap riveted joint is to be designed for a steel bracket as shown in Fig. 9.24.

The bracket plate is 25 mm thick. All rivets are to be of the same size. Load on the bracket, $P = 50 \text{kN};$ rivet spacing, $C = 100 \text{mm};$ load arm, $e = 400 \text{mm}.$

Permissible shear stress is 65 MPa and crushing stress is 120 MPa. Determine the size of the rivets to be used for the joint.

Solution. Given: $t = 25 \text{mm} ; \ P = 50 \text{kN} = 50 \times 10^3 \text{N} ; \ e = 400 \text{mm} ; \ n = 7 ; \ \tau = 65 \text{MPa} \ = 65 \text{N/mm}^2 ; \ \sigma_c = 120 \text{MPa} = 120 \text{N/mm}^2$

First of all, let us find the centre of gravity ($G$) of the rivet system.

Let $\bar{x} =$ Distance of centre of gravity from $OY,$

$\bar{y} =$ Distance of centre of gravity from $OX.$
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\[ x_1, x_2, x_3... = \text{Distances of centre of gravity of each rivet from } OY, \]
\[ y_1, y_2, y_3... = \text{Distances of centre of gravity of each rivet from } OX. \]

We know that
\[
x = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7}{n} = \frac{100 + 200 + 200 + 200}{7} = 100 \text{ mm} \quad (\because x_1 = x_6 = x_7 = 0)
\]

and
\[
y = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7}{n} = \frac{200 + 200 + 200 + 100 + 100}{7} \approx 114.3 \text{ mm} \quad (\because y_5 = y_6 = 0)
\]

∴ The centre of gravity \((G)\) of the rivet system lies at a distance of 100 mm from \(OY\) and 114.3 mm from \(OX\), as shown in Fig. 9.25.

We know that direct shear load on each rivet,

\[ P_s = \frac{P}{n} = \frac{50 \times 10^3}{7} = 7143 \text{ N} \]

The direct shear load acts parallel to the direction of load \(P\), i.e. vertically downward as shown in Fig. 9.25.

Turning moment produced by the load \(P\) due to eccentricity \((e)\)

\[ = P \times e = 50 \times 10^3 \times 400 = 20 \times 10^6 \text{ N-mm} \]

This turning moment is resisted by seven rivets as shown in Fig. 9.25.

Let \(F_1, F_2, F_3, F_4, F_5, F_6\) and \(F_7\) be the secondary shear load on the rivets 1, 2, 3, 4, 5, 6 and 7 placed at distances \(l_1, l_2, l_3, l_4, l_5, l_6\) and \(l_7\) respectively from the centre of gravity of the rivet system as shown in Fig. 9.26.
From the geometry of the figure, we find that
\[ l_1 = l_3 = \sqrt{(100)^2 + (200 - 114.3)^2} = 131.7 \text{ mm} \]
\[ l_2 = 200 - 114.3 = 85.7 \text{ mm} \]
\[ l_4 = l_7 = \sqrt{(100)^2 + (114.3 - 100)^2} = 101 \text{ mm} \]
and
\[ l_5 = l_6 = \sqrt{(100)^2 + (114.3)^2} = 152 \text{ mm} \]

Now equating the turning moment due to eccentricity of the load to the resisting moment of the rivets, we have

\[ P \times e = \frac{F_1}{l_1} \left[ (l_1)^2 + (l_3)^2 + (l_4)^2 + (l_7)^2 + (l_5)^2 + (l_6)^2 \right] \]
\[ = \frac{F_1}{l_1} \left[ 2(l_1)^2 + (l_2)^2 + 2(l_4)^2 + 2(l_5)^2 \right] \]

\[ ... (\because l_1 = l_3, l_4 = l_7 \text{ and } l_5 = l_6) \]

\[ 50 \times 10^3 \times 400 = \frac{F_1}{131.7} \left[ 2(131.7)^2 + (85.7)^2 + 2(101)^2 + 2(152)^2 \right] \]
\[ 20 \times 10^6 \times 131.7 = F_1(34690 + 7345 + 20402 + 46208) = 108645 F_1 \]
\[ \therefore F_1 = 20 \times 10^6 \times 131.7 / 108645 = 24244 \text{ N} \]

Since the secondary shear loads are proportional to their radial distances from the centre of gravity, therefore

\[ F_2 = F_1 \times \frac{l_2}{l_1} = 24244 \times \frac{85.7}{131.7} = 15776 \text{ N} \]
\[ F_3 = F_1 \times \frac{l_3}{l_1} = F_1 = 24244 \text{ N} \]
\[ F_4 = F_1 \times \frac{l_4}{l_1} = 24244 \times \frac{101}{131.7} = 18593 \text{ N} \]

Note: This picture is given as additional information and is not a direct example of the current chapter.
By drawing the direct and secondary shear loads on each rivet, we see that the rivets 3, 4 and 5 are heavily loaded. Let us now find the angles between the direct and secondary shear load for these three rivets. From the geometry of Fig. 9.26, we find that

\[
\cos \theta_3 = \frac{100}{131.7} = 0.76
\]

\[
\cos \theta_4 = \frac{100}{101} = 0.99
\]

and

\[
\cos \theta_5 = \frac{100}{152} = 0.658
\]

Now resultant shear load on rivet 3,

\[
R_3 = \sqrt{(P_3)^2 + (F_3)^2 + 2 P_3 \times F_3 \times \cos \theta_3}
\]

\[
= \sqrt{(7143)^2 + (24 244)^2 + 2 \times 7143 \times 24 244 \times 0.76} = 30 033 N
\]

Resultant shear load on rivet 4,

\[
R_4 = \sqrt{(P_4)^2 + (F_4)^2 + 2 P_4 \times F_4 \times \cos \theta_4}
\]

\[
= \sqrt{(7143)^2 + (18 593)^2 + 2 \times 7143 \times 18 593 \times 0.99} = 25 684 N
\]

and resultant shear load on rivet 5,

\[
R_5 = \sqrt{(P_5)^2 + (F_5)^2 + 2 P_5 \times F_5 \times \cos \theta_5}
\]

\[
= \sqrt{(7143)^2 + (27 981)^2 + 2 \times 7143 \times 27 981 \times 0.658} = 33 121 N
\]

The resultant shear load may be determined graphically, as shown in Fig. 9.26.

From above we see that the maximum resultant shear load is on rivet 5. If \(d\) is the diameter of rivet hole, then maximum resultant shear load \((R_5)\),

\[
33 121 = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times d^2 \times 65 = 51 d^2
\]

\[
\therefore \quad d^2 = 33 121 / 51 = 649.4 \quad \text{or} \quad d = 25.5 \text{ mm}
\]

From Table 9.7, we see that according to IS : 1929–1982 (Reaffirmed 1996), the standard diameter of the rivet hole \((d)\) is 25.5 mm and the corresponding diameter of rivet is 24 mm.

Let us now check the joint for crushing stress. We know that

\[
\text{Crushing stress} = \frac{\text{Max. load}}{\text{Crushing area}} = \frac{R_5}{d \times t} = \frac{33 121}{25.5 \times 25} = 51.95 \text{ N/mm}^2 = 51.95 \text{ MPa}
\]

Since this stress is well below the given crushing stress of 120 MPa, therefore the design is satisfactory.
Example 9.15. The bracket as shown in Fig. 9.27, is to carry a load of 45 kN. Determine the size of the rivet if the shear stress is not to exceed 40 MPa. Assume all rivets of the same size.

Solution. Given: \( P = 45 \text{ kN} = 45 \times 10^3 \text{ N} ; \tau = 40 \text{ MPa} = 40 \text{ N/mm}^2 ; e = 500 \text{ mm} ; n = 9 \)

First of all, let us find the centre of gravity of the rivet system. Since all the rivets are of same size and placed symmetrically, therefore the centre of gravity of the rivet system lies at \( G \) (rivet 5) as shown in Fig. 9.28.

We know that direct shear load on each rivet,

\[
P_s = \frac{P}{n} = \frac{45 \times 10^3}{9} = 5000 \text{ N}
\]

The direct shear load acts parallel to the direction of load \( P \), i.e. vertically downward as shown in the figure.

Turning moment produced by the load \( P \) due to eccentricity \( e \)

\[
= Pe = 45 \times 10^3 \times 500 = 22.5 \times 10^6 \text{ N-mm}
\]

This turning moment tends to rotate the joint about the centre of gravity \( G \) of the rivet system in a clockwise direction. Due to this turning moment, secondary shear load on each rivet is produced. It may be noted that rivet 5 does not resist any moment.

Let \( F_1, F_2, F_3, F_4, F_5, F_6, F_7 \) and \( F_9 \) be the secondary shear load on rivets 1, 2, 3, 4, 6, 7, 8 and 9 at distances \( l_1, l_2, l_3, l_4, l_6, l_7, l_8 \) and \( l_9 \) from the centre of gravity \( G \) of the rivet system as shown in Fig. 9.28. From the symmetry of the figure, we find that

\[
l_1 = l_3 = l_7 = l_9 = \sqrt{(100)^2 + (120)^2} = 156.2 \text{ mm}
\]

Now equating the turning moment due to eccentricity of the load to the resisting moments of the rivets, we have

\[
P \times e = \frac{F_1}{l_1} \left[ (l_1)^2 + (l_2)^2 + (l_3)^2 + (l_4)^2 + (l_6)^2 + (l_7)^2 + (l_8)^2 + (l_9)^2 \right]
\]
The secondary shear loads acts perpendicular to the line joining the centre of rivet and the centre of gravity (G), as shown in Fig. 9.28 and their direction is clockwise.

By drawing the direct and secondary shear loads on each rivet, we see that the rivets 3, 6 and 9 are heavily loaded. Let us now find the angle between the direct and secondary shear loads for these rivets. From the geometry of the figure, we find that

\[
\cos \theta_3 = \cos \theta_9 = \frac{100}{l_3} = \frac{100}{156.2} = 0.64
\]

\[
\therefore \text{Resultant shear load on rivets 3 and 9,}
R_3 = R_9 = \sqrt{(P_3)^2 + (P_3)^2 + 2 \times P_3 \times F_3 \times \cos \theta_3}
\]

\[
= \sqrt{(5000)^2 + (23120)^2 + 2 \times 5000 \times 23120 \times 0.64} = 26600 \text{ N}
\]

\[
\text{(}\because \ l_3 = l_9 \text{ and } \cos \theta_3 = \cos \theta_9\text{)}
\]

and resultant shear load on rivet 6,

\[
R_6 = P_x + F_6 = 5000 + 14800 = 19800 \text{ N}
\]

The resultant shear load \((R_3 \text{ or } R_9)\) may be determined graphically as shown in Fig. 9.28. From above we see that the maximum resultant shear load is on rivets 3 and 9.

If \(d\) is the diameter of the rivet hole, then maximum resultant shear load \((R_3)\),

\[
d^2 = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times d^2 \times 40 = 31.42 \ d^2
\]

\[
\therefore \ 
26600 = 26600 / 31.42 = 846 \text{ or } \ d = 29 \text{ mm}
\]

From Table 9.7, we see that according to IS : 1929 – 1982 (Reaffirmed 1996), the standard diameter of the rivet hole \((d)\) is 29 mm and the corresponding diameter of the rivet is 27 mm. \textbf{Ans.}

**Example 9.16.** Find the value of \(P\) for the joint shown in Fig. 9.29 based on a working shear stress of 100 MPa for the rivets. The four rivets are equal, each of 20 mm diameter.

**Solution.** Given : \(\tau = 100 \text{ MPa} = 100 \text{ N/mm}^2\); \(n = 4\); \(d = 20 \text{ mm}\)

We know that the direct shear load on each rivet,

\[
P_x = \frac{P}{n} = \frac{P}{4} = 0.25 \ P
\]

The direct shear load on each rivet acts in the direction of the load \(P\), as shown in Fig. 9.30. The centre of gravity of the rivet group will lie at \(E\) (because of symmetry). From Fig. 9.30, we find that
the perpendicular distance from the centre of gravity \( E \) to the line of action of the load (or eccentricity),

\[ EC = e = 100 \text{ mm} \]

\[ \therefore \] Turning moment produced by the load at the centre of gravity \((E)\) of the rivet system due to eccentricity

\[ = P_e = P \times 100 \text{ N-mm (anticlockwise)} \]

This turning moment is resisted by four rivets as shown in Fig. 9.30. Let \( F_A, F_B, F_C \) and \( F_D \) be the secondary shear load on the rivets, \( A, B, C, \) and \( D \) placed at distances \( l_A, l_B, l_C \) and \( l_D \) respectively from the centre of gravity of the rivet system.

From Fig. 9.30, we find that

\[ l_A = l_D = 200 + 100 = 300 \text{ mm} \; \text{and} \; l_B = l_C = 100 \text{ mm} \]

We know that

\[ P \times e = \frac{F_A}{l_A} \left[ (l_A)^2 + (l_B)^2 + (l_C)^2 + (l_D)^2 \right] = \frac{F_A}{l_A} \left[ 2(l_A)^2 + 2(l_B)^2 \right] \]

\[ \therefore \] \( P \times 100 = \frac{F_A}{300} \left[ 2 \times (300)^2 + 2 \times (100)^2 \right] = \frac{2000}{3} \times F_A \)

\[ \therefore \]

\[ F_A = P \times 100 \times 3 / 2000 = 0.15 \text{ P N} \]

Since the secondary shear loads are proportional to their radial distances from the centre of gravity, therefore

\[ F_B = F_A \times \frac{l_B}{l_A} = \frac{3 \times P}{20} \times \frac{100}{300} = 0.05 \text{ P N} \]

\[ F_C = F_A \times \frac{l_C}{l_A} = \frac{3 \times P}{20} \times \frac{100}{300} = 0.05 \text{ P N} \]

and

\[ F_D = F_A \times \frac{l_D}{l_A} = \frac{3 \times P}{20} \times \frac{300}{300} = 0.15 \text{ P N} \]

The secondary shear loads on each rivet act at right angles to the lines joining the centre of the rivet to the centre of gravity of the rivet system as shown in Fig. 9.30.

Now let us find out the resultant shear load on each rivet. From Fig. 9.30, we find that

Resultant load on rivet \( A \),

\[ R_A = P_s - F_A = 0.25 \text{ P} - 0.15 \text{ P} = 0.10 \text{ P} \]
Resultant load on rivet $B$,

\[ R_B = P_s - F_B = 0.25P - 0.05P = 0.20P \]

Resultant load on rivet $C$,

\[ R_C = P_s + F_C = 0.25P + 0.05P = 0.30P \]

and resultant load on rivet $D$,

\[ R_D = P_s + F_D = 0.25P + 0.15P = 0.40P \]

From above we see that the maximum shear load is on rivet $D$. We know that the maximum shear load ($R_D$),

\[ 0.40P = \frac{\pi}{4}d^2 \tau = \frac{\pi}{4}(20)^2 100 = 31420 \]

\[ \therefore P = 31420 / 0.40 = 78550 \text{ N} = 78.55 \text{ kN} \quad \text{Ans.} \]

**Example 9.17.** A bracket is riveted to a column by 6 rivets of equal size as shown in Fig. 9.31. It carries a load of 60 kN at a distance of 200 mm from the centre of the column. If the maximum shear stress in the rivet is limited to 150 MPa, determine the diameter of the rivet.

**Solution.** Given : $n = 6$ ; $P = 60 \text{ kN} = 60 \times 10^3 \text{ N}$ ; $e = 200 \text{ mm}$ ; $\tau = 150 \text{ MPa} = 150 \text{ N/mm}^2$

Since the rivets are of equal size and placed symmetrically, therefore the centre of gravity of the rivet system lies at $G$ as shown in Fig. 9.32. We know that ditect shear load on each rivet,

\[ P_s = \frac{P}{n} = \frac{60 \times 10^3}{6} = 10000 \text{ N} \]

Let $F_1$, $F_2$, $F_3$, $F_4$, $F_5$ and $F_6$ be the secondary shear load on the rivets 1, 2, 3, 4, 5 and 6 at distances $l_1$, $l_2$, $l_3$, $l_4$, $l_5$ and $l_6$ from the centre of gravity ($G$) of the rivet system. From the symmetry of the figure, we find that

\[ l_1 = l_3 = l_5 = l_6 = \sqrt{(75)^2 + (50)^2} = 90.1 \text{ mm} \]

and

\[ l_2 = l_4 = 50 \text{ mm} \]
Now equating the turning moment due to eccentricity of the load to the resisting moments of the rivets, we have

\[ P \times e = \frac{F_1}{l_1} \left[ (l_2)^2 + (l_3)^2 + (l_4)^2 + (l_5)^2 + (l_6)^2 \right] \]

\[ = \frac{F_1}{l_1} \left[ 4(l_1)^2 + 2(l_2)^2 \right] \]

\[ \therefore \quad 60 \times 10^3 \times 200 = \frac{F_1}{90.1} \left[ 4(90.1)^2 + 2(50)^2 \right] = 416 \ F_1 \]

or

\[ F_1 = 60 \times 10^3 \times 200 / 416 = 28 846 \ N \]

Since the secondary shear loads are proportional to the radial distances from the centre of gravity, therefore

\[ F_2 = F_1 \times \frac{l_1}{l_1} = 28 846 \times \frac{50}{90.1} = 16 008 \ N \]

\[ F_3 = F_1 \times \frac{l_1}{l_1} = F_1 = 28 846 \ N \quad \ldots (\because l_3 = l_1) \]

\[ F_4 = F_1 \times \frac{l_1}{l_1} = F_1 = 28 846 \ N \quad \ldots (\because l_4 = l_1) \]

\[ F_5 = F_1 \times \frac{l_1}{l_1} = F_1 = 28 846 \ N \quad \ldots (\because l_5 = l_1) \]

and

\[ F_6 = F_1 \times \frac{l_1}{l_1} = F_1 = 28 846 \ N \quad \ldots (\because l_6 = l_1) \]

By drawing the direct and secondary shear loads on each rivet, we see that the rivets 1, 2 and 3 are heavily loaded. Let us now find the angles between the direct and secondary shear loads for these three rivets. From the geometry of the figure, we find that

\[ \cos \theta_1 = \cos \theta_3 = \frac{50}{l_1} = \frac{50}{90.1} = 0.555 \]
Resultant shear load on rivets 1 and 3,

\[ R_1 = R_3 = \sqrt{(P_s)^2 + (F_1)^2 + 2P_s \times F_1 \times \cos \theta_1} \]

\[ \vdots (F_1 = F_3 \text{ and } \cos \theta_1 = \cos \theta_3) \]

\[ = \sqrt{(10000)^2 + (28846)^2 + 2 \times 10000 \times 28846 \times 0.555} \]

\[ = \sqrt{100 \times 10^6 + 832 \times 10^6 + 320 \times 10^6} = 35348 \text{ N} \]

and resultant shear load on rivet 2,

\[ R_2 = P_s + F_2 = 10000 + 16008 = 26008 \text{ N} \]

From above we see that the maximum resultant shear load is on rivets 1 and 3. If \( d \) is the diameter of rivet hole, then maximum resultant shear load (\( R_1 \) or \( R_3 \)),

\[ d^2 = \frac{35384}{117.8} = 300.4 \text{ or } d = 17.33 \text{ mm} \]

From Table 9.7, we see that according to IS : 1929 – 1982 (Reaffirmed 1996), the standard diameter of the rivet hole (\( d \)) is 19.5 mm and the corresponding diameter of the rivet is 18 mm. **Ans.**

**Example 9.18.** A bracket in the form of a plate is fitted to a column by means of four rivets A, B, C and D in the same vertical line, as shown in Fig. 9.33. \( AB = BC = CD = 60 \text{ mm} \). E is the mid-point of BC. A load of 100 kN is applied to the bracket at a point F which is at a horizontal distance of 150 m from E. The load acts at an angle of 30° to the horizontal. Determine the diameter of the rivets which are made of steel having a yield stress in shear of 240 MPa. Take a factor of safety of 1.5.

What would be the thickness of the plate taking an allowable bending stress of 125 MPa for the plate, assuming its total width at section ABCD as 240 mm?

**Solution.** Given : \( n = 4 \); \( AB = BC = CD = 60 \text{ mm} \); \( P = 100 \text{ kN} = 100 \times 10^3 \text{ N} \); \( EF = 150 \text{ mm} \); \( \theta = 30^\circ \); \( \tau_y = 240 \text{ MPa} = 240 \text{ N/mm}^2 \); \( F.S. = 1.5 \); \( \sigma_b = 125 \text{ MPa} = 125 \text{ N/mm}^2 \); \( b = 240 \text{ mm} \)

**Solution.**

Let \( d = \) Diameter of rivets.

We know that direct shear load on each rivet,

\[ P_s = \frac{P}{n} = \frac{100 \times 10^3}{4} = 25000 \text{ N} \]
The direct shear load on each rivet acts in the direction of 100 kN load (i.e. at 30° to the horizontal) as shown in Fig. 9.34. The centre of gravity of the rivet group lies at E. From Fig. 9.34, we find that the perpendicular distance from the centre of gravity E to the line of action of the load (or eccentricity of the load) is

\[ EG = e = EF \sin 30° = 150 \times \frac{1}{2} = 75 \text{ mm} \]

\[ ∴ \text{ Turning moment produced by the load } P \text{ due to eccentricity} \]

\[ = Pe = 100 \times 10^3 \times 75 = 7500 \times 10^3 \text{ N-mm} \]

This turning moment is resisted by four bolts, as shown in Fig. 9.34. Let \( F_A, F_B, F_C \) and \( F_D \) be the secondary shear load on the rivets, \( A, B, C, \) and \( D \) placed at distances \( l_A, l_B, l_C \) and \( l_D \) respectively from the centre of gravity of the rivet system.

From Fig. 9.34, we find that

\[ l_A = l_D = 60 + 30 = 90 \text{ mm and } l_B = l_C = 30 \text{ mm} \]

We know that

\[ P \times e = \frac{F_A}{l_A} [(l_A)^2 + (l_B)^2 + (l_C)^2 + (l_D)^2] = \frac{F_A}{l_A} [2(l_A)^2 + 2(l_B)^2] \]

\[ = \frac{F_A}{90} [2(90)^2 + 2(30)^2] = 200 \ F_A \]

\[ \therefore F_A = 7500 \times 10^3 / 200 = 37500 \text{ N} \]

Since the secondary shear loads are proportional to their radial distances from the centre of gravity, therefore,

\[ F_B = F_A \times \frac{l_B}{l_A} = 37500 \times \frac{30}{90} = 12500 \text{ N} \]

\[ F_C = F_A \times \frac{l_C}{l_A} = 37500 \times \frac{30}{90} = 12500 \text{ N} \]

and

\[ F_D = F_A \times \frac{l_D}{l_A} = 37500 \times \frac{90}{90} = 37500 \text{ N} \]

Now let us find the resultant shear load on each rivet.

From Fig. 9.34, we find that angle between \( F_A \) and \( P_s = θ_A = 150° \)

Angle between \( F_B \) and \( P_s = θ_B = 150° \)

Angle between \( F_C \) and \( P_s = θ_C = 30° \)

Angle between \( F_D \) and \( P_s = θ_D = 30° \)

\[ ∴ \text{ Resultant load on rivet } A, \]

\[ R_A = \sqrt{(P_A)^2 + (F_A)^2 + 2P_A \times F_A \times \cos θ_A} \]

\[ = \sqrt{(25000)^2 + (37500)^2 + 2 \times 25000 \times 37500 \times \cos 150°} \]

\[ = \sqrt{625 \times 10^6 + 1406 \times 10^6 - 1623.8 \times 10^6} = 15492 \text{ N} \]
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Resultant shear load on rivet B,
\[ R_B = \sqrt{(P_s)^2 + (F_B)^2 + 2 P_s \times F_B \times \cos \theta_B} \]
\[ = \sqrt{(25 000)^2 + (12 500)^2 + 2 \times 25 000 \times 12 500 \times \cos 150^\circ} \]
\[ = \sqrt{625 \times 10^6 + 156.25 \times 10^6 - 541.25 \times 10^6} = 15 492 \text{ N} \]

Resultant shear load on rivet C,
\[ R_C = \sqrt{(P_s)^2 + (F_C)^2 + 2 P_s \times F_C \times \cos \theta_C} \]
\[ = \sqrt{(25 000)^2 + (12 500)^2 + 2 \times 25 000 \times 12 500 \times \cos 30^\circ} \]
\[ = \sqrt{625 \times 10^6 + 156.25 \times 10^6 + 541.25 \times 10^6} = 36 366 \text{ N} \]

and resultant shear load on rivet D,
\[ R_D = \sqrt{(P_s)^2 + (F_D)^2 + 2 P_s \times F_D \times \cos \theta_D} \]
\[ = \sqrt{(25 000)^2 + (37 500)^2 + 2 \times 25 000 \times 37 500 \times \cos 30^\circ} \]
\[ = \sqrt{625 \times 10^6 + 1406 \times 10^6 + 1623.8 \times 10^6} = 60 455 \text{ N} \]

The resultant shear load on each rivet may be determined graphically as shown in Fig. 9.35.

From above we see that the maximum resultant shear load is on rivet D. We know that maximum resultant shear load \((R_D)\),
\[ 60 455 = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times d^2 \times \frac{\tau}{F.S.} \]
\[ = \frac{\pi}{4} \times d^2 \times \frac{240}{1.5} = 125.7 \times d^2 \]
\[ \therefore d^2 = \frac{60 455}{125.7} = 481 \]
or
\[ d = 21.9 \text{ mm} \]

From Table 9.7, we see that the standard diameter of the rivet hole \((d)\) is 23.5 mm and the corresponding diameter of rivet is 22 mm. **Ans.**

**Thickness of the plate**

Let
\[ t = \text{Thickness of the plate in mm}, \]
\[ \sigma_b = \text{Allowable bending stress for the plate} = 125 \text{ MPa} = 125 \text{ N/mm}^2 \]
\[ b = \text{Width of the plate} = 240 \text{ mm} \]

Consider the weakest section of the plate (i.e. the section where it receives four rivet holes of diameter 23.5 mm and thickness \(t\) mm) as shown in Fig. 9.36. We know that moment of inertia of the plate about \(X-X\),
\[ I_{XX} = \text{M.I. of solid plate about } X-X - ^4\text{M.I. of 4 rivet holes about } X-X \]

\[ = \text{M.I. of 4 rivet holes about their centroidal axis} + 2 A(h_1)^2 + 2 A(h_2)^2 \]
\[ = \text{Area of rivet hole} \]
\[ = \text{Area of rivet hole} \]
Bending moment,
\[ M = P \times e = 100 \times 10^3 \times 75 \]
\[ = 7500 \times 10^3 \text{ N-mm} \]

Distance of neutral axis \((X-X)\) from the top most fibre of the plate,
\[ y = \frac{b}{2} = \frac{240}{2} = 120 \text{ mm} \]

We know that
\[ \frac{M}{I} = \frac{\sigma_b}{y} \]
\[ \frac{7500 \times 10^3}{724674 t} = 125 \]
\[ \text{or} \]
\[ \frac{10.35}{t} = 1.04 \]
\[ \text{or} \]
\[ \frac{10.35}{1.04} = 9.95 \text{ say 10 mm} \]

Ans.

**EXERCISES**

1. A single riveted lap joint is made in 15 mm thick plates with 20 mm diameter rivets. Determine the strength of the joint, if the pitch of rivets is 60 mm. Take \(\sigma_t = 120 \text{ MPa}; \tau = 90 \text{ MPa} \) and \(\sigma_c = 160 \text{ MPa} \).
   [Ans. 28 280 N]

2. Two plates 16 mm thick are joined by a double riveted lap joint. The pitch of each row of rivets is 90 mm. The rivets are 25 mm in diameter. The permissible stresses are as follows:
   \(\sigma_t = 140 \text{ MPa} ; \tau = 110 \text{ MPa} \) and \(\sigma_c = 240 \text{ MPa} \).
   Find the efficiency of the joint.
   [Ans. 53.5%]

3. A single riveted double cover butt joint is made in 10 mm thick plates with 20 mm diameter rivets with a pitch of 60 mm. Calculate the efficiency of the joint, if
   \(\sigma_t = 100 \text{ MPa} ; \tau = 80 \text{ MPa} \) and \(\sigma_c = 160 \text{ MPa} \).
   [Ans. 53.8%]

4. A double riveted double cover butt joint is made in 12 mm thick plates with 18 mm diameter rivets. Find the efficiency of the joint for a pitch of 80 mm, if
   \(\sigma_t = 115 \text{ MPa} ; \tau = 80 \text{ MPa} \) and \(\sigma_c = 160 \text{ MPa} \).
   [Ans. 62.6%]

5. A double riveted lap joint with chain riveting is to be made for joining two plates 10 mm thick. The allowable stresses are : \(\sigma_t = 60 \text{ MPa} ; \tau = 50 \text{ MPa} \) and \(\sigma_c = 80 \text{ MPa} \). Find the rivet diameter, pitch of rivets and distance between rows of rivets. Also find the efficiency of the joint.
   [Ans. \(d = 20 \text{ mm} ; p = 73 \text{ mm} ; p_b = 38 \text{ mm} ; \eta = 71.7\%\)]

6. A triple riveted lap joint with zig-zag riveting is to be designed to connect two plates of 6 mm thickness. Determine the dia. of rivet, pitch of rivets and distance between the rows of rivet. Indicate how the joint will fail. Assume : \(\sigma_t = 120 \text{ MPa} ; \tau = 100 \text{ MPa} \) and \(\sigma_c = 150 \text{ MPa} \).
   [Ans. \(d = 14 \text{ mm} ; p = 78 \text{ mm} ; p_b = 35.2 \text{ mm} \)]

7. A double riveted butt joint, in which the pitch of the rivets in the outer rows is twice that in the inner rows, connects two 16 mm thick plates with two cover plates each 12 mm thick. The diameter of rivets is 22 mm. Determine the pitches of the rivets in the two rows if the working stresses are not to exceed the following limits:
Tensile stress in plates = 100 MPa; Shear stress in rivets = 75 MPa; and bearing stress in rivets and plates = 150 MPa.

Make a fully dimensioned sketch of the joint by showing at least two views.

[Ans. 107 mm, 53.5 mm]

8. Design a double riveted double strap butt joint for the longitudinal seam of a boiler shell, 750 mm in diameter, to carry a maximum steam pressure of 1.05 N/mm² gauge. The allowable stresses are:

\[ \sigma_t = 35 \text{ MPa}; \quad \tau = 28 \text{ MPa} \quad \text{and} \quad \sigma_c = 52.5 \text{ MPa} \]

Assume the efficiency of the joint as 75%.

[Ans. \( t = 16 \text{ mm}; d = 25 \text{ mm}; p = 63 \text{ mm}; \quad \rho_b = 37.5 \text{ mm}; \quad t_1 = t_2 = 10 \text{ mm}; \quad m = 37.5 \text{ mm} \)]

9. Design a triple riveted double strap butt joint with chain riveting for a boiler of 1.5 m diameter and carrying a pressure of 2.4 N/mm². The inside diameter of the drum is 1.3 metres. The joint is to be designed for a steam pressure of 2.4 N/mm². The working stresses to be used are:

\[ \sigma_t = 105 \text{ MPa}; \quad \tau = 77 \text{ MPa} \quad \text{and} \quad \sigma_c = 162.5 \text{ MPa} \]

[Ans. \( d = 20 \text{ mm}; p = 50 \text{ mm} \)]

10. Design a triple riveted longitudinal double strap butt joint with unequal straps for a boiler. The inside diameter of the longest course of the drum is 1.3 metres. The joint is to be designed for a steam pressure of 2.4 N/mm². The working stresses to be used are:

\[ \sigma_t = 77 \text{ MPa}; \quad \tau = 62 \text{ MPa} \quad \text{and} \quad \sigma_c = 120 \text{ MPa} \]

Assume the efficiency of the joint as 81%.

[Ans. \( t = 26 \text{ mm}; d = 31.5 \text{ mm}; p = 200 \text{ mm}; \quad t_1 = 19.5 \text{ mm}; \quad t_2 = 16.5 \text{ mm}; \quad m = 47.5 \text{ mm} \)]

11. Design the longitudinal and circumferential joint for a boiler whose diameter is 2.4 metres and is subjected to a pressure of 1 N/mm². The longitudinal joint is a triple riveted butt joint with an efficiency of about 85% and the circumferential joint is a double riveted lap joint with an efficiency of about 70%. The pitch in the outer rows of the rivets is to be double than in the inner rows and the width of the cover plates is unequal. The allowable stresses are:

\[ \sigma_t = 77 \text{ MPa}; \quad \tau = 56 \text{ MPa} \quad \text{and} \quad \sigma_c = 120 \text{ MPa} \]

Assume that the resistance of rivets in double shear is 1.875 times that of single shear. Draw the complete joint.

12. A triple riveted butt joint with equal double cover plates (zig-zag riveting) is used for the longitudinal joint of a Lancashire boiler of 2.5 m internal diameter. The working steam pressure is 1.12 N/mm² and the efficiency of the joint is 85 per cent. Calculate the plate thickness for mild steel of 460 MPa ultimate tensile strength. Assume ratio of tensile to shear stresses as 7/6 and factor of safety 4. The resistance of the rivets in double shear is to be taken as 1.875 times that of single shear. Design a suitable circumferential joint also.

[Ans. \( n = 5; \eta = 88\% \)]

13. Two lengths of mild steel flat tie bars 200 mm × 10 mm are to be connected by a double riveted double cover butt joint, using 24 mm diameter rivets. Design the joint, if the allowable working stresses are 112 MPa in tension, 84 MPa in shear and 200 MPa in crushing.

[Ans. \( n = 6; \eta = 90\% \)]

14. Two mild steel tie bars for a bridge structure are to be joined by a double cover butt joint. The thickness of the tie bar is 20 mm and carries a tensile load of 400 kN. Design the joint if the allowable stresses are:

\[ \sigma_t = 90 \text{ MPa}; \quad \tau = 75 \text{ MPa} \quad \text{and} \quad \sigma_c = 150 \text{ MPa} \]

Assume the strength of rivet in double shear to be 1.75 times that of in single shear.

[Ans. \( b = 150 \text{ mm}; d = 27 \text{ mm}; n = 6; \eta = 90\% \)]

15. Two lengths of mild steel tie rod having width 200 mm are to be connected by means of Lozenge joint with two cover plates to withstand a tensile load of 180 kN. Completely design the joint, if the permissible stresses are 80 MPa in tension; 65 MPa in shear and 160 MPa in crushing. Draw a neat sketch of the joint.

[Ans. \( t = 13 \text{ mm}; d = 22 \text{ mm}; n = 5; \eta = 86.5\% \)]

16. A bracket is supported by means of 4 rivets of same size, as shown in Fig. 9.37. Determine the diameter of the rivet if the maximum shear stress is 140 MPa.

[Ans. 16 mm]
17. A bracket is riveted to a column by 6 rivets of equal size as shown in Fig. 9.38. It carries a load of 100 kN at a distance of 250 mm from the column. If the maximum shear stress in the rivet is limited to 63 MPa, find the diameter of the rivet. 

[Ans. 41 mm]

18. A bracket in the form of a plate is fitted to a column by means of four rivets of the same size, as shown in Fig. 9.39. A load of 100 kN is applied to the bracket at an angle of 60° to the horizontal and the line of action of the load passes through the centre of the bottom rivet. If the maximum shear stress for the material of the rivet is 70 MPa, find the diameter of rivets. What will be the thickness of the plate if the crushing stress is 100 MPa? 

[Ans. 29 mm; 1.5 mm]

QUESTIONS

1. What do you understand by the term riveted joint? Explain the necessity of such a joint.
2. What are the various permanent and detachable fastenings? Give a complete list with the different types of each category.
3. Classify the rivet heads according to Indian standard specifications.
4. What is the material used for rivets?
5. Enumerate the different types of riveted joints and rivets.
6. What is an economical joint and where does it find applications?
7. What is the difference between caulking and fullering? Explain with the help of neat sketches.
8. Show by neat sketches the various ways in which a riveted joint may fail.
9. What do you understand by the term ‘efficiency of a riveted joint’? According to I.B.R., what is the highest efficiency required of a riveted joint?
10. Explain the procedure for designing a longitudinal and circumferential joint for a boiler.
11. Describe the procedure for designing a lozenge joint.
12. What is an eccentric riveted joint? Explain the method adopted for designing such a joint?

OBJECTIVE TYPE QUESTIONS

1. A rivet is specified by
   (a) shank diameter  
   (b) length of rivet  
   (c) type of head  
   (d) length of tail
2. The diameter of the rivet hole is usually .......... the nominal diameter of the rivet.
   (a) equal to  (b) less than  (c) more than

3. The rivet head used for boiler plate riveting is usually
   (a) snap head  (b) pan head
   (c) counter sunk head  (d) conical head

4. According to Unwin’s formula, the relation between diameter of rivet hole \( d \) and thickness of plate \( t \) is given by
   (a) \( d = t \)  (b) \( d = 1.6 \sqrt{t} \)
   (c) \( d = 2t \)  (d) \( d = 6t \)
   where \( d \) and \( t \) are in mm.

5. A line joining the centres of rivets and parallel to the edge of the plate is known as
   (a) back pitch  (b) marginal pitch
   (c) gauge line  (d) pitch line

6. The centre to centre distance between two consecutive rivets in a row, is called
   (a) margin  (b) pitch
   (c) back pitch  (d) diagonal pitch

7. The objective of caulking in a riveted joint is to make the joint
   (a) free from corrosion  (b) stronger in tension
   (c) free from stresses  (d) leak-proof

8. A lap joint is always in ......shear.
   (a) single  (b) double

9. A double strap butt joint (with equal straps) is
   (a) always in single shear  (b) always in double shear
   (c) either in single shear or double shear  (d) any one of these

10. Which of the following riveted butt joints with double straps should have the highest efficiency as per Indian Boiler Regulations?
    (a) Single riveted  (b) Double riveted
    (c) Triple riveted  (d) Quadruple riveted

11. If the tearing efficiency of a riveted joint is 50%, then ratio of diameter of rivet hole to the pitch of rivets is
    (a) 0.20  (b) 0.30
    (c) 0.50  (d) 0.60

12. The strength of the unriveted or solid plate per pitch length is equal to
    (a) \( p \times d \times \sigma_t \)  (b) \( p \times t \times \sigma_t \)
    (c) \( (p - d) \times d \times \sigma_t \)  (d) \( (p - d) \times t \times \sigma_t \)

13. The longitudinal joint in boilers is used to get the required
    (a) length of boiler  (b) diameter of boiler
    (c) length and diameter of boiler  (d) efficiency of boiler

14. For longitudinal joint in boilers, the type of joint used is
    (a) lap joint with one ring overlapping the other  (b) butt joint with single cover plate
    (c) butt joint with double cover plates  (d) any one of these

15. According to Indian standards, the diameter of rivet hole for a 24 mm diameter of rivet, should be
    (a) 23 mm  (b) 24 mm
    (c) 25 mm  (d) 26 mm

**ANSWERS**

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